

A LESLIE-GOWER TYPE PREDATION MODEL WITH NON-MONOTONIC FUNCTIONAL RESPONSE AND STRONG ALLEE EFFECT ON PREY

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ABSTRACT

In this work, a modified predator-prey model is analyzed, considering the Allee effect. To describe the dynamics of the model, we demonstrate the existence of a positively invariant region, the boundedness, and the permanence of the trajectories. We provide necessary and sufficient conditions for the existence and explicit form of up to two positive equilibria. One equilibrium is always a hyperbolic saddle, while the other can be an attractor, repeller, or weak focus. Additionally, we find two key scenarios: (i) a separatrix curve on the phase plane dividing the behavior of trajectories into qualitatively distinct regions, and (ii) a homoclinic curve generated by the stable and unstable manifolds of a saddle point in the interior of the first quadrant. These structures highlight the system's sensitivity to initial conditions, particularly near the separatrix. Bifurcations can occur in the system, including Hopf bifurcations, which further influence the model's dynamics. Finally, numerical simulations are presented to validate the analytical results.

Keywords Predator-prey model · Leslie-Gower · Strong Allee efect · Nonmonotonic functional response · Equilibrium analysis

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