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ON THE METHOD OF TRANSFORMING A NONLOCAL BOUNDARY VALUE PROBLEM INTO A CAUCHY PROBLEM FOR A SECOND ORDER ORDINARY DIFFERENTIAL EQUATION

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ABSTRACT

Finding solutions to problems with non-local boundary condition, even for in the simple of ordinary differential equations or systems of equations, requires special research. In this work we will study the following equation

$$\ell[y] \equiv y''(x) + ay'(x) + by(x) = 0, \quad 0 < x < 1, \tag{1}$$

to the subject to linearly independent nonlocal boundary conditions with constant coefficients

$$L[u] \equiv \alpha_{k0}y(0) + \alpha_{k1}y(1) + \beta_{k0}y'(0) + \beta_{k1}y'(1) = \gamma_k, \quad (k = 1, 2). \tag{2}$$

Here a, b, α_{kj}, β_{kj} and γ_k, $(k = 1, 2; j = 0, 1)$ are given real constants.
For problem (1), (2) the following as called a basic relation

$$y'(0)Z(-\xi) - y(0)Z'(-\xi) - y'(1)Z(1-\xi) + y(1)Z'(1-\xi) + ay(0)Z(-\xi) - ay(1)Z(1-\xi) =$$
$$= \begin{cases} y(\xi), & \xi \in (0,1), \\ \frac{1}{2}y(\xi), & \xi = 0,\ \xi = 1 \end{cases} \tag{3}$$

is found. $Z(\xi)$ denotes the fundamental solution of the adjoint equation corresponding to (1) in the sense of Lagrange. From (3) we get two necessary conditions as follows

$$\frac{1}{2}y(0) = y'(0)Z(0) - y(0)Z'(0) - y'(1)Z(1) + y(1)Z'(1) + ay(0)Z(0) - ay(1)Z(1), \tag{4}$$

$$\frac{1}{2}y'(0) = y'(1)Z'(1) - y'(0)Z'(0) + by(1)Z(1) - by(0)Z'(0) \tag{5}$$

The following theorems are proved:

Theorem 1. If a and b are given real constants, then any solution of equation (1) satisfies to linear independent necessary conditions (4) and (5).

Theorem 2. Let us assume that a, b, α_{kj}, β_{kj} and γ_k, $(k = 1, 2; j = 0, 1)$ are given real constants and the boundary conditions (2) are linearly independent. Under certain conditions, problem (1), (2) is reduced to the Cauchy problem.

Keywords Nonlocal boundary condition · First and second basic relation · Necessary conditions

References

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