



# A FRACTIONAL DIFFERENTIAL EQUATION FRAMEWORK FOR HOUSEHOLD CONSUMPTION EXPENDITURE DETERMINATION

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## ABSTRACT

Household consumption expenditure constitutes a central pillar of aggregate demand and serves as a key indicator of economic welfare, macroeconomic stability, and living standards. Because consumption accounts for a substantial share of gross domestic product in most economies, understanding its dynamic behavior is essential for effective fiscal and monetary policy design. Traditional models of consumption are typically formulated using integer-order differential equations and are largely grounded in classical economic theories. While these models provide valuable baseline insights, they often fall short in capturing several empirically observed features of consumer behavior, such as long-memory effects, habit formation, persistence in spending patterns, and delayed adjustment to changes in income, wealth, or policy conditions. To address these limitations, this study proposes a fractional-order dynamic framework for modeling household consumption expenditure using tools from fractional calculus. By introducing non-integer-order derivatives into the consumption dynamics, the model explicitly incorporates memory and path dependence, allowing current consumption decisions to depend not only on present economic conditions but also on past consumption trajectories. This approach offers a more faithful representation of real-world consumption behavior, where adjustment processes are gradual and influenced by accumulated experiences and expectations. A governing fractional differential equation describing the temporal evolution of household consumption is formulated and solved systematically using the Laplace transform and its inverse. The analytical solution provides clear insights into the role of the fractional order in shaping adjustment speed and persistence. The stability properties and long-run behavior of the model are then examined, revealing conditions under which consumption converges to a steady state or exhibits prolonged transitional dynamics. The results demonstrate that fractional-order dynamics yield greater flexibility and realism than classical models, particularly in capturing sluggish responses and long-memory effects. The study concludes by discussing policy implications and outlining potential directions for future research, including empirical calibration and extensions to heterogeneous household settings.

**Keywords** Household consumption · fractional calculus · memory effects · income dynamics · economic modeling

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