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A MOVING-SERVER $M/M/1$ QUEUE WITH ZONE-INDUCED INTERRUPTIONS: ERGODIC CONDITION AND CLOSED-FORM STATIONARY DISTRIBUTION

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ABSTRACT

Motivation. Mobile edge platforms—high-speed trains, autonomous vessels, low-altitude drones—serve users only while inside radio-coverage zones; traffic queues up during shadowed stretches. Proper buffer sizing therefore demands a queueing model that couples mobility with intermittent service availability.

Model. Let $\{N_t\}_{t\geq 0}$ be the queue length and $S_t \in \{0,1\}$ indicate whether the server is inside a coverage zone (1) or not (0). Zone lengths are independent exponentials: $\mathrm{Exp}(\alpha)$ for coverage, $\mathrm{Exp}(\beta)$ for out-of-coverage; thus (N_t, S_t) is a continuous-time Markov chain with infinitesimal generator

$$Q = \begin{pmatrix} Q_{11} & Q_{10} \\ Q_{01} & Q_{00} \end{pmatrix}, \qquad Q_{ss'} = \big(q_{ss'}(n,m)\big)_{n,m\geq 0},$$

where $q_{11}(n, n-1) = \mu$, $q_{11}(n, n+1) = \lambda$, $q_{10}(n,n) = \beta$, $q_{01}(n,n) = \alpha$, and all other transition rates are 0. The chain is stochastically equivalent to an $M/M/1$ queue whose server *breaks* with rate α and *repairs* with rate β.

Proposition 1 (Ergodicity). Define $\rho := \lambda(\alpha+\beta)/(\beta\mu)$. The chain is positive recurrent, i.e. the queue is ergodic, *iff*

$$\lambda \;<\; \frac{\beta\mu}{\alpha+\beta} \quad\iff\quad 0<\rho<1.$$

Proof sketch. A Foster–Lyapunov function $V(n,s) = n + \gamma s$ with suitably chosen $\gamma > 0$ yields $\langle QV, (n,s)\rangle \leq -\varepsilon$ outside a finite set; thus the drift criterion holds.

Proposition 2 (Stationary Distribution). Assuming $0 < \rho < 1$, the joint stationary probabilities are

$$\pi_{n,1} = \frac{(\lambda+\beta)(1-\rho)}{\lambda+\beta+\alpha}\,\rho^n, \qquad \pi_{n,0} = \frac{\alpha(1-\rho)}{\lambda+\beta+\alpha}\,\rho^n, \quad n\geq 0,$$

so the marginal distribution of N is geometric with parameter $1-\rho$. Expected queue length and waiting time follow in closed form:

$$\mathbb{E}[N] = \frac{\rho}{1-\rho}, \qquad \mathbb{E}[W] = \frac{\rho}{\mu(1-\rho)}.$$

The *physical* parameters—mean coverage length $1/\alpha$, mean shadow length $1/\beta$, and vehicle speed v—enter performance metrics only through α and β. Consequently, increasing coverage density (larger β) or vehicle speed (scaling both rates) widens the stability region linearly. Because probabilities decay geometrically, backlog targets such as $\Pr\{N > k\} \leq \varepsilon$ translate into explicit admissible arrival rates without simulation, enabling real-time buffer sizing for mobile edge platforms.

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