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FINITE GENERATED CLASSES IN k-VALUED LOGIC WITHOUT MAJORITY AND CHOICE FUNCTIONS

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ABSTRACT

This paper investigates the class of functions in k-valued logic, focusing on finite classes that do not contain majority and choice functions. Specifically, we analyze the class $F \subseteq P_k$, where P_k represents the set of all k-valued logic functions. A function class F is termed *structurally finite* if it can be generated from a finite set of base functions. We demonstrate that the class F, satisfying condition $0x$, is finitely generated and does not include majority or choice functions.

Our primary result establishes that the structural finiteness of the class F implies that all functions within it depend on at most $k-1$ variables. Formally, we show that:

$$F \subseteq T_0 \quad \text{where} \quad T_0 = \{f \in P_k \mid f(x_1, \ldots, x_n) = 0 \text{ if } x_i = 0\}.$$

Additionally, the absence of majority and choice functions in F is proven using the following result:

$$\mu \notin F, \quad \text{where} \quad \mu(x_1, x_2, \ldots, x_n) = \begin{cases} 1, & \text{if the majority of } x_i = 1, \\ 0, & \text{otherwise,} \end{cases}$$

and similarly for choice functions.

These results have significant implications in areas such as network traffic optimization and cellular systems (LTE, 5G), where reducing computational complexity and improving algorithmic efficiency are crucial. The established framework generalizes classical Boolean logic results to k-valued logic, with potential applications in the optimization of multi-valued logic systems [1, 2, 3, 4, 5].

***K*eywords** k-valued logic · Majority functions · Choice functions · Structural finiteness · Network optimization

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