

DIFFERENTIAL OPERATOR-BASED IMPUTATION: A ROBUST METHOD FOR INCOMPLETE DATA ANALYSIS

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ABSTRACT

When only some data points are available in a dataset, imputation of the approximate values at unknown points often becomes essential to maintain the integrity and usability of the data. This need becomes especially critical in real-world applications where unobserved or missing data are prevalent and can severely impact the performance of statistical analyses and machine learning models. To address this issue, this study introduces a novel imputation technique that reconstructs missing data points using only the known entries, without requiring any prior assumption about the underlying data distribution or functional form.

The method is based on three fundamental equations derived using approximate differential operators. These equations consist of the central difference and the three-point forward and backward approximate differential operators. The value at an unknown point is computed using these equations by incorporating the neighboring data points to its left and right.

To rigorously assess the accuracy, stability, and generalization capacity of the proposed approach, synthetic datasets are generated from various analytical functions that exhibit different behaviors within a fixed domain. These datasets simulate distinct imputation scenarios under controlled conditions, providing a robust testing environment. Comparative analyses with state-of-the-art imputation methods are performed using standard error metrics, including Mean Absolute Error (MAE), Mean Relative Error (MRE), and Root Mean Square Error (RMSE).

Experimental results show that the proposed method consistently delivers high-precision imputations, even when the proportion of missing data is considerable. The local nature of the computation, which is based solely on the nearest neighbors, enhances both efficiency and interpretability. Furthermore, the use of approximate differential operators contributes to the numerical reliability and transparency of the method, making it particularly well suited for critical applications involving incomplete or noisy data.

Keywords Missing Data Imputation · Approximate Differential Operators · Local Estimation

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