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SOME NORMAL APPROXIMATIONS FOR THE VARIANCE OF A RESIDUAL WAITING TIME PROCESS

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ABSTRACT

The residual waiting time process is a fundamental quantity in renewal theory and represents the time remaining from an arbitrary observation epoch until the next renewal event. Its mean and variance are important performance measures in queueing, reliability, and inventory systems; however, they are generally difficult to evaluate because the associated renewal function rarely admits a closed-form expression. This study develops computationally tractable normal approximations for the mean and variance of the residual waiting time process when the interarrival times follow a Weibull distribution with shape parameter $\beta \geq 3$. In this parameter range, the Weibull distribution becomes less skewed and can be effectively approximated by a normal distribution with matching first two moments.

Let $U(t)$ denote the renewal function and $W(t)$ the residual waiting time. By Wald's identity, $\mathbb{E}[W(t)] = \mu U(t) - t$, where μ is the mean interarrival time. Using a truncated normal series representation for $U(t)$ developed by Cui and Xie (2003), we provided three approximations for $E(W(t))$. Their accuracy is assessed through Monte Carlo simulation for Weibull interarrival times. The numerical results show close agreement with the simulated values.

The analysis is further extended to the variance of $W(t)$. For validation, an explicit variance formula is derived for Erlang$(3, \lambda)$ interarrival times, and the case $\lambda = 1$ is examined numerically using Simpson's rule for the integral term. The variance approximations become highly accurate as t increases, with relative errors below 1% for $t \geq 6$, and converge to the equilibrium variance $8/3$. The results demonstrate that the proposed method provides a simple, efficient, and accurate alternative for evaluating the first two moments of residual waiting time when direct renewal calculations are unavailable.

Keywords Residual waiting time · Renewal function · Normal approximation · Weibull distribution · Erlang distribution

References

[1] Cui L. and Xie M., Some normal approximations for renewal function of large Weibull shape parameter, *Communications in Statistics – Simulation and Computation*, 32(1): 1–16, 2003.

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