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ON SEMIGROUPS OF TRANSFORMATIONS WHOSE RESTRICTIONS ARE ELEMENTS OF A GIVEN SEMIGROUP

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ABSTRACT

Let $n \in \mathbb{Z}^+$ and let $X_n = \{1, \ldots, n\}$. As it is well known that P_n, T_n, I_n and S_n denote the partial transformation semigroup, (full) transformation semigroup, symmetric inverse semigroup (the semigroup of all partial one to one maps) and symmetric group (group of all permutations) on the set X_n, respectively. Now, let Y be a non-empty subset of the set X_n with cardinality m $(1 \leq m \leq n-1)$. Without loss of generality, we can consider the set X_m, rather than Y and define the semigroups

$$\begin{aligned}
IS_{(n,m)} &= \{\, \alpha \in I_n : \alpha_{|_{X_m}} \in S_m \,\}, & II_{(n,m)} &= \{\, \alpha \in I_n : \alpha_{|_{X_m}} \in I_m \,\},\\
TT_{(n,m)} &= \{\, \alpha \in T_n : \alpha_{|_{X_m}} \in T_m \,\}, & PS_{(n,m)} &= \{\, \alpha \in P_n : \alpha_{|_{X_m}} \in S_m \,\},\\
PT_{(n,m)} &= \{\, \alpha \in P_n : \alpha_{|_{X_m}} \in T_m \,\}, & PI_{(n,m)} &= \{\, \alpha \in P_n : \alpha_{|_{X_m}} \in I_m \,\},\\
PP_{(n,m)} &= \{\, \alpha \in P_n : \alpha_{|_{X_m}} \in P_m \,\}
\end{aligned}$$

for $1 \leq m \leq n-1$. In this study, we introduce and examine these new semigroups.

Keywords Partial (Full) transformation semigroup · Symmetric inverse semigroup · Symmetric group

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