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DISTRIBUTION OF FEASIBLE SOLUTIONS FOR APPROXIMATE BLOCK DIAGONALIZATION WITH PENALTY COEFFICIENTS IN CLASSICAL, PSEUDO-QUANTUM AND QUANTUM ANNEALING

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ABSTRACT

One example of matrix computations, which is very important in science and engineering, is eigenpair computation. Recently, the Block Jacobi method, which possesses excellent parallelism, has attracted considerable attention as one of the good methods for calculation of eigenpairs. The method divides the input matrix into square blocks and repeatedly annihilates the off-diagonal blocks by orthogonal transformations to gradually bring the matrix to a diagonal form [1]. It has been confirmed that the convergence of the method can be accelerated by introducing preprocessing known as Approximate Block Diagonalization (ABD), which maximizes the sum of squared Frobenius norms of diagonal blocks by symmetric permutation of rows and columns, which does not change the eigenvalues of the target matrix [2].

Since ABD is a combinatorial optimization problem, a recent research attempted to solve it by quantum annealing [3]. In general, annealing methods cannot deal with constraints directly, so the penalty method is used to incorporate the constraints. However, determining the best penally parameter is not easy, especially when there are two or more constraints as in the ABD problem. In this research, in addition to quantum annealing, we applied classical annealing and pseudo-quantum annealing to the ABD problem and investigated the influence of the two penalty parameters by grid search. In particular, we focused on two kind of indices, namely, the number of feasible solutions and the maximum objective function value, as functions of the two parameters and studied how they exhibit different behaviors among the three annealing methods. Finally, we propose a method to select the penalty parameters automatically for the three annealing methods.

Keywords Block diagonalization · Annealing · QUBO · Unconstrained optimization · Penalty method · automatic parameter tuning

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