
BINET-FIBONACCI CALCULUS AND $N = 2$ SUPERSYMMETRIC GOLDEN QUANTUM OSCILLATOR

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ABSTRACT

The Binet-Fibonacci calculus, as $\varphi\varphi'$ - two base quantum calculus, relates Fibonacci derivative with Binet formula of Fibonacci number operator, acting in Fock space of quantum states. It provides a tool to study the Golden oscillator with energy spectrum in form of Fibonacci numbers. Here we generalize this model to supersymmetric number operator and corresponding Binet formula for supersymmetric Fibonacci operator \mathcal{F}_N . It determines the Hamiltonian of supersymmetric Golden oscillator, acting in $H_f \otimes H_F$ - fermion-boson Hilbert space and belonging to $N = 2$ supersymmetric algebra. Trace on fermions of this model reduces the Hamiltonian to the Golden oscillator. The eigenstates of the super Fibonacci number are double degenerate and can be characterized by a point of the super-Bloch sphere. By the supersymmetric Fibonacci annihilation operator, we construct the coherent states as eigenstates of this operator. Entanglement of fermions with bosons in these states is calculated by the concurrence, represented by the Gram determinant and Fibonacci exponential functions. These functions have been appeared as descriptive for inner product of the Golden coherent states in Fock-Bargmann representation. The reference state, coming from the limit $\alpha \rightarrow 0$ and corresponding von Neumann entropy, measuring fermion-boson entanglement, are characterized by the Golden ratio.

Keywords Fibonacci numbers · Golden Ratio · supersymmetry · coherent states · Golden oscillator

References

- [1] Pashaev O.K, and Nalci S., Golden quantum oscillator and Binet-Fibonacci calculus, J. Phys. A: Math. Theor., 45: 015303, 2012.
- [2] Pashaev O.K, Two-circles theorem, q-periodic functions and entangled qubit states, J. Phys.: Conference Series, 482: 012033, 2014.
- [3] Pashaev O.K, Variations on a theme of q-oscillator, Phys. Scr. 90: 074010, 2015.
- [4] Pashaev O.K, Quantum calculus of Fibonacci divisors and infinite hierarchy of bosonic-fermionic Golden quantum oscillator, Int. J. Geom. Methods in Modern Physics . 18: 2150075, 2021.
- [5] Pashaev O.K, The Bell-based super-coherent states: Uncertainty relations, Golden ratio and fermion-boson entanglement, Int. J. Geom. Methods in Modern Physics . <https://dx.doi.org/10.1142/S0219887824502670>, 2024