V International Conference on Mathematics and its Applications in Science and Engineering
(ICMASE 2024)

ICMASE 2024

FIBONACCI AND LUCAS HYBRID SPLIT QUATERNIONS

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ABSTRACT

In this research, we begin by providing a concise overview of split quaternions and their generalizations involving complex, dual, and hyperbolic numbers. Subsequently, we introduce a new generalization called hybrid split quaternions and present their characteristics, including conjugate and norm. Furthermore, we explore their applications to Fibonacci and Lucas number sequences, along with providing generating functions and Binet's formula specific to Fibonacci hybrid split and Lucas hybrid split quaternions.

To understand the gap that this study aims to fill in the literature, we will summarize the existing research on split quaternions. Following this, we will present a table to thoroughly examine the primary objective of this study.

Split quaternions extend the algebra of standard quaternions by incorporating both real and hyperbolic components. While classical quaternions consist of four elements, including one scalar and three vector components, split quaternions are characterized by two real and two hyperbolic components. For real numbers z_i, a split quaternion can be expressed as $Q = z_0 + z_1 i + z_2 j + z_3 k$. In this formulation, the hyperbolic units j and k have the property that their squares equal 1. Split quaternions are particularly useful in physics and engineering, with applications in contexts such as Lorentz transformations and special relativity. The set of split quaternions is formally defined as:

$$\mathbb{H}_s = \left\{ Q = z_0 + z_1 i + z_2 j + z_3 k : i^2 = -1, j^2 = k^2 = 1, ijk = 1,\ z_i \in \mathbb{R} \right\}$$

The units i, j, and k exhibit non-commutative behavior, similar to that of the real quaternion units. However, split quaternions introduce unique algebraic characteristics, including the presence of nilpotent elements, zero divisors, and non-trivial idempotents [1, 2, 14].

Extensions of split quaternions are obtained by changing the component of the units. Lets have a split quaternion Q as $Q = z_0 + z_1 i + z_2 j + z_3 k$. If coefficients z_i are in complex number, dual number, and hyperbolic number then Q is called as complex split quaternions ($\mathbb{H}_s^{\mathbb{C}}$), dual split quaternions ($\mathbb{H}_s^{\mathbb{D}}$), hyperbolic split quaternions ($\mathbb{H}_s^{\mathbb{H}}$), respectively. This can be summarized with the following table.

Split Quaternions	Representation	Coefficients	References
$\mathbb{H}_s^{\mathbb{C}}$	$Q = Q_a + \mathbf{i} Q_b$	$z_i \in \mathbb{C}$	[8]
$\mathbb{H}_s^{\mathbb{D}}$	$Q = Q_a + \varepsilon Q_b$	$z_i \in \mathbb{D}$	[3, 4, 9, 12, 16]
$\mathbb{H}_s^{\mathbb{H}}$	$Q = Q_a + \mathbf{h} Q_b$	$z_i \in \mathbb{H}$	[6, 7, 15, 17]
		$z_i \in \mathbb{K}$	-

Table 1: Extensions of Split Quaternions

In the later sections of the study, fundamental information on the original components will be presented, and split hybrid quaternions will be defined. This will enable us to examine the Fibonacci and Lucas number sequences within the established framework.

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Other studies related to the topic examined in our research, which we recommend to the reader, include [5, 13, 10, 11].

***K*eywords** Hybrid quaternions · Split quaternions · Fibonacci sequences

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