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AN INVERSE STEFAN-TYPE PROBLEM FOR AN ANTI-PARABOLIC EQUATION WITH DOUBLE FREE BOUNDARIES

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ABSTRACT

Free boundary problems arise in many areas of fluid dynamics, where it is necessary to determine simultaneously both the solution and either a part of the unknown domain or the boundary itself. The problem becomes more complex when the process is described by anti-parabolic equations in a domain D with two unknown boundaries. In this study the following problem

$$\frac{\partial u(t,x)}{\partial t} + \frac{\partial^2 u(t,x)}{\partial x^2}, \quad (t,x) \in D, \tag{1}$$

$$u(0,x) = \varphi(x), \quad x \in [0,\ell], \quad t > 0, \tag{2}$$

$$u\left(t, \gamma_k(t)\right) = \psi_k(t), \quad t \geq 0, \quad k = 1,2, \tag{3}$$

$$\frac{\partial u(t,x)}{\partial x}\Big|_{x=\gamma_k(t)}, \quad t \geq 0, \quad k = 1,2, \tag{4}$$

is considered. Here $\varphi(x)$, $x \in [0,\ell]$, and $\psi_k(t)$, $t > 0$ are given continuous functions, while $u(t,x)$ and $\gamma_k(t)$, $t \geq 0$, $(k = 1,2)$ are the unknown functions.

The novelty of this work is the constructive reduction of the double free boundary problem to a system of second-kind Volterra integral equations for the unknowns $u(x,t)$ and $\gamma_k(t)$, $(k = 1,2)$, providing a unified analytical framework for the simultaneous determination of the solution and both moving boundaries.

Keywords Free boundary problem · Anti-parabolic equation · Inverse Stefan problem

References

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