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BIFURCATION STRUCTURES AND CHAOTIC DYNAMICS IN A FRACTIONAL-ORDER RICHARDS MODEL WITH ENVIRONMENTAL FEEDBACK

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ABSTRACT

In this study, a fractional-order population model based on the modified Richards growth law is proposed to investigate the combined effects of environmental feedback and the Allee effect on population dynamics. The model incorporates a density-dependent carrying capacity and a critical population threshold, providing a more realistic description of ecological growth processes. Since low-dimensional continuous and fractional-order systems cannot generate chaotic behavior, the piecewise constant argument (PCA) approach is employed to derive a discrete-time counterpart of the proposed model.

The existence and local stability of the equilibrium points are analyzed by means of the Schur–Cohn criterion. The results show that the discrete model exhibits a Flip bifurcation near the positive equilibrium point E_1 and a Neimark–Sacker bifurcation near the positive equilibrium point E_2 as the system parameters pass through the corresponding critical thresholds. The direction and stability of the bifurcating solutions are determined using center manifold theory and normal form analysis. Numerical bifurcation diagrams and phase portraits reveal the emergence of periodic, quasi-periodic, and chaotic dynamics. The presence of chaos is further confirmed through the computation of the maximum Lyapunov exponents.

To suppress undesirable chaotic oscillations, state-feedback and hybrid control strategies are implemented. Numerical simulations demonstrate that both control approaches successfully stabilize the system and enlarge the parameter regions corresponding to regular dynamics. The obtained results indicate that environmental feedback and the Allee effect significantly influence the stability boundaries and bifurcation structure of the model, leading to a rich variety of dynamical outcomes.

***K*eywords** Bifurcation · Chaos · Mathematical model · Stability · Fractional order

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