
PROPER RANDOM WALK SPLINE MODELS

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ABSTRACT

Splines are central to nonparametric regression, and intrinsic spline formulations such as the second-order random walk (rw2) have become standard for smoothing structured data. However, their intrinsic nature entails specific structural features—non-constant marginal variances, inflated boundary uncertainty, sensitivity to grid design, and unbounded predictive variance—that arise from the absence of a finite correlation range. While these characteristics serve the purpose of flexible smoothing, they can lead to unstable or misleading behavior in prediction or sparse-data settings when the intrinsic nature of the model is not carefully accounted for.

This thesis introduces the PRW2, a full-rank Gaussian Markov random field that serves as a proper counterpart to the intrinsic RW2 formulation. PRW2 preserves RW2's smoothing strength and computational efficiency while providing a coherent probabilistic structure with constant marginal variance, bounded predictive uncertainty, and stability under domain changes. Although RW2 and PRW2 belong to different model spaces, PRW2 closely reproduces the smoothing behavior of RW2 as the range parameter increases, ensuring smoothing behavior without requiring intrinsic specification.

. Bayesian Statistics · Gaussian Markov Random Fields (GMRF) · INLA · Spatio-temporal modeling

Keywords Spline smoothing · INLA · Spatio-temporal modeling · Gaussian Markov Random Fields (GMRF) · Bayesian Statistics

References

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