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# BOUNDEDNESS AND MAPPING PROPERTIES OF COMMUTATORS OF $B$ -MAXIMAL OPERATORS

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Esra Kaya \*

*Bilecik Şeyh Edebali University, Faculty of Science, Department of Mathematics, Bilecik, TURKEY*

## ABSTRACT

This study focuses on the contemporary harmonic analysis associated with the generalized translation operator and the Laplace-Bessel differential operator within the framework of variable exponent Lebesgue spaces. In recent years, non-standard function spaces, particularly variable Lebesgue spaces, have attracted significant interest due to their applications in fluid mechanics, elasticity, and variational calculus. In this work, we investigate the structural and mapping properties of maximal sublinear operators generated by the Laplace-Bessel operator. Specifically, we establish the boundedness of both the commutator of the  $B$ -maximal operator and the  $B$ -maximal commutator itself on the variable exponent Lebesgue spaces  $L_{p(\cdot),\gamma}(\mathbb{R}_{k,+}^n)$ . Under appropriate regularity conditions on the variable exponent  $p(\cdot)$ , such as the standard local log-Hölder continuity and suitable decay conditions at infinity, and assuming that the local symbol function  $b$  belongs to the generalized Bounded Mean Oscillation space  $BMO_\gamma$ , we prove the continuous embedding and robust boundedness results for these non-linear operator structures. Furthermore, the corresponding sharp  $B$ -maximal operator is introduced and evaluated to provide a complete characterization of the underlying space geometry. The pointwise estimates and innovative decomposition techniques utilized in our proofs allow us to overcome the difficulties arising from the non-invariant nature of the variable exponent under the action of the generalized translation operator  $T^y$ . Our findings provide a comprehensive, natural, and non-trivial extension of classical maximal commutator estimates to the variable exponent Laplace-Bessel setting, thereby filling a significant gap in the weighted operator theory on non-standard spaces.

**Keywords** Bounded mean oscillation space · commutator · maximal operator · variable Lebesgue space

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\*Corresponding Author's E-mail: [esra.kaya@bilecik.edu.tr](mailto:esra.kaya@bilecik.edu.tr)