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NEW FIXED POINT RESULTS FOR GENERALIZED (ψ, θ, L) -TYPE $\mathcal{R}$ - CONTRACTIVE MAPPINGS IN $\mathcal{F}$-METRIC SPACES WITH AN APPLICATION

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ABSTRACT

The goal of this paper states a fixed point theorem for generalized (ψ, θ, L)-type $\mathcal{R}$-contractive mappings in $\mathcal{F}$-metric spaces having a binary relation. This analysis is based on a relation theoretic approach, where the condition of contraction needs to hold only for those elements that are related by the relation $\mathcal{R}$. The main condition of contraction can be described by means of an altering distance function, a control function and an extra perturbation term with $N_T(x, y)$. As the classical triangle inequality does not hold in $\mathcal{F}$-metric spaces, a Picard admissible condition is defined along with the Picard orbit to cope with the mixed distance term in $M_T(x, y)$. The existence of fixed point under conditions of $\mathcal{R}$ -completeness, (T)-closedness of the relation $\mathcal{R}$, $\mathcal{R}$ -continuity, and an appropriate condition for the auxiliary function is demonstrated. A result on the uniqueness of the fixed point is obtained under the comparability of fixed points through the binary relation. This theorem includes Banach-type and generalized $\mathcal{R}$ -contractive type results. As an application, the proposed fixed point theorem is used to study a Caputo-type fractional initial value problem. The fractional differential equation is reformulated as an equivalent integral equation, and the corresponding integral operator is examined using the existing fixed point methodology. The necessary conditions for the existence and uniqueness of a solution are determined. Moreover, an Ulam–Hyers stability result is achieved under a Lipschitz-type assumption. The example shows how relation-theoretic fixed point methods in $\mathcal{F}$-metric spaces can be utilized to study the stability analysis of nonlinear fractional models.

***K*eywords** Fixed point; $\mathcal{F}$-metric space; binary relation;$\mathcal{R}$ -contractive mapping; fractional differential equation; Ulam–Hyers stability

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