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ACCELERATING THE SOLUTION OF MULTIPLE SHIFTED SKEW-SYMMETRIC SYSTEMS VIA RANDOMIZED DEFLATION PRECONDITIONING

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ABSTRACT

We consider the efficient solution of multiple shifted skew-symmetric linear systems

$$(\alpha_i I + S)x_i = b_i, \quad i = 1, \ldots, m,$$

where all systems share the same skew-symmetric matrix S but differ in the shift parameters α_i. Such systems arise in a variety of scientific and engineering applications. When some shift parameters are small, the corresponding systems become highly ill-conditioned, making iterative solvers expensive. Solving each shifted system independently often leads to substantial computational redundancy, since the systems share the same skew-symmetric matrix S and exhibit closely related spectral structures. To exploit this property, we propose a randomized deflation preconditioning framework for multiple shifted skew-symmetric systems. The proposed method constructs a low-dimensional approximation of the dominant invariant subspace only once by applying a randomized Nyström approximation. This spectral information is then reused to efficiently build deflation preconditioners for all shifted systems with negligible additional cost. We derive theoretical bounds for the effective condition numbers of the preconditioned systems. Numerical experiments on large-scale problems demonstrate that the proposed approach significantly accelerates GMRES convergence and substantially reduces the overall setup cost compared with constructing separate preconditioners for individual shifts.

Keywords multiple shifted skew-symmetric linear systems · randomized deflation preconditioning · Nyström approximation · GMRES

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