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NONLOCAL FISHER-KPP EQUATION WITH FREE BOUNDARIES: NUMERICAL STRATEGIES FOR ACCELERATED SPREADING

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ABSTRACT

The nonlocal Fisher-KPP equation with free boundaries models population dynamics in which individuals undergo long-range dispersal across a habitat whose edges evolve in response to population flux. Solving this problem numerically is non-trivial: the coupling of a nonlocal integral operator with an explicitly moving domain precludes standard parabolic solvers and demands specialized discretization strategies. We propose and compare two numerical strategies. The Front-Tracking Runge-Kutta (FTRK) method operates directly on the physical domain, coupling fourth-order Runge-Kutta time integration with dynamic grid adaptation to preserve spatial resolution during significant domain expansion. The Front-Fixing (FF) method applies a Landau-type coordinate transformation that maps the evolving domain onto a fixed computational interval, converting the governing equation into a partial integro-differential equation (PIDE) with state-dependent advection terms; these are discretized using upwind and Lax-Wendroff schemes paired with Simpson's quadrature for the integral operator. While FTRK accommodates domain growth naturally at the cost of a progressively larger system, FF maintains a fixed number of degrees of freedom at the price of decreasing effective spatial resolution in regimes of strong expansion. Both methods are validated against theoretical criteria for the spreading-vanishing dichotomy and correctly reproduce accelerated propagation driven by fat-tailed dispersal kernels.

Keywords Nonlocal diffusion · Free boundary problem · Front-tracking method · Front-fixing method · Accelerated spreading

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