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# EXPLICIT GREEN FUNCTION REPRESENTATIONS FOR DIRICHLET PROBLEMS IN COMPLEX POLYDOMAINS

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## ABSTRACT

Boundary value problems for complex partial differential equations play a fundamental role in complex analysis and its applications, yet many aspects remain open in the framework of several complex variables. In this work, we investigate the Dirichlet boundary value problem for homogeneous and inhomogeneous complex partial differential equations in polydomains, with particular emphasis on explicit analytic constructions based on Green function representations.

The classical theory of the Dirichlet problem in one complex variable provides powerful tools through integral representations; however, extending these techniques to multidimensional product domains introduces additional analytical difficulties arising from the geometry of polydomain boundaries and the interaction between multiple complex variables. To address these challenges, we develop a Green function approach adapted to polydomains, allowing the derivation of explicit representation formulas for solutions.

Using suitably constructed Green functions, we establish integral representation formulas for holomorphic or complex-valued solutions satisfying prescribed boundary data. These representations provide a unified framework for treating both homogeneous and inhomogeneous equations and lead naturally to necessary and sufficient solvability conditions for the Dirichlet problem. The method highlights the role of boundary symmetries and enables a systematic extension of classical potential-theoretic techniques to higher-dimensional complex settings.

Furthermore, the transition from the unit disc to polydomains is analyzed in detail, demonstrating how multidimensional effects influence the structure of Green kernels and the associated boundary integral operators. Despite the increased complexity, the resulting solvability theory preserves strong analogies with the one-variable case and yields well-posed formulations without imposing excessively restrictive assumptions.

The results contribute to the development of Green function methods in several complex variables and provide a constructive analytical framework for the study of Dirichlet-type boundary value problems in multidimensional complex domains.

***Keywords*** Polydomain · Dirichlet Problem · Green function

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