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# FINITE-TIMELESS LOCAL OBSERVABILITY FOR LINEAR CONTROL SYSTEMS ON LIE GROUPS

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## ABSTRACT

Observability problem is one of the classical fundamental problem in the area and it is important from the applications point of view. The aim of this paper is to introduce a new approach to the observability problem in control theory. In this study, our approach comes from the finite-time point of view. For a general control system  $\Sigma = (G, \mathcal{D}, h, V)$  on Lie groups, where  $G$  and  $V$  are Lie groups,  $\mathcal{D}$  is the dynamic of the system and  $h$  is a smooth function between  $G$  and  $V$ , observability problem is to distinguish the points of the state space and the solutions of the system just by looking their image under  $h$  in  $V$  with the positive time.

In [1, 2], the authors study observability problem of linear control systems on Lie groups. In [1], the authors characterize local and global observability with a Lie group homomorphism and in [2], the authors characterize observability with a kind of projection. In [3], the author examines the existence of observable linear control systems on Lie groups appears in [1] and [2]. Recently, in [4], the authors give a comparison the observability properties of the class of linear control systems in two different manifolds: on the Euclidean space  $\mathbb{R}^n$  in a more general setup, on a connected Lie group  $G$ . Related to these articles, a new time approach for the usual local observability is given.

In this study, linear control systems on connected Lie groups has taken into consideration and finite-timeless observability is introduced. For this new concept of observability, almost indistinguishability is defined and it has been shown that this is an equivalence relation. Also, properties of the image of almost indistinguishable points from the neutral element has studied. Finally, local observability is associated with finite-timeless local observability.

**Keywords** observability · indistinguishability · linear control systems · Lie groups · Lie algebras

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