GEOMETRIC STRUCTURE-PRESERVING METHODS FOR TWO-DIMENSIONAL STOCHASTIC NONLINEAR WAVE EQUATIONS

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ABSTRACT

This report proposes a novel structure-preserving numerical framework for two-dimensional stochastic nonlinear wave equations driven by Stratonovich-type multiplicative noise under periodic boundary conditions. Stochastic nonlinear wave equations are foundational mathematical models utilized across various physical systems, including electromagnetics, elasticity, and fluid dynamics, where wave propagation is inevitably influenced by external random perturbations and environmental uncertainties. Since these systems exhibit rich geometric structures within the Hamiltonian framework, conventional numerical methods often introduce unphysical artifacts, artificial energy dissipation, or phase errors during long-time simulations. This motivates the development of structure-preserving numerical schemes capable of accurately capturing the long-time dynamics of stochastic wave propagation.

To preserve the geometric structure, the target equation is reformulated as a multi-symplectic Hamiltonian system with inherent multi-symplectic and momentum conservation properties. While previous literature explored multi-symplectic techniques using radial basis functions [1] or discontinuous Galerkin methods [2], the application of spectral element approaches remains scarce. For spatial semi-discretization, the authors employ the Galerkin spectral element method (GSEM). Drawing inspiration from historical discrete Galerkin projections [3], this approach effectively combines the high accuracy of spectral methods with the geometric flexibility of finite elements.

For temporal discretization, the Crank-Nicolson method is applied. Crucially, a specially designed structure-preserving discretization technique is introduced for the nonlinear terms. This ensures that the resulting fully discrete numerical method rigorously preserves the discrete multi-symplectic structure and conserves momentum. Furthermore, a rigorous convergence analysis is performed in the L^2-norm, establishing an analytical error bound. Finally, two-dimensional numerical experiments validate the theoretical analysis and demonstrate the outstanding capability of the proposed method in maintaining physical invariants over long-time simulations.

***K*eywords** Galerkin spectral element method · Preserving structure · Error estimate

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