SPECTRAL ANALYSIS OF REACTIONDIFFUSION SYSTEMS VIA PINNS

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ABSTRACT

Reactiondiffusion systems are widely used to describe dynamic behaviors and pattern formation in fields such as physics, chemistry, and biology. A key part of understanding these systems is analyzing the stability of steady-state solutions, which typically involves solving an eigenvalue problem derived from a linearization of the governing equations. However, exact solutions to such problems are often out of reachparticularly when the system includes multiple components or when the operators involved are not self-adjoint. In these cases, researchers turn to numerical methods as the most practical means of performing spectral analysis and exploring the systems behavior. In this study, we explore the use of Physics-Informed Neural Networks (PINNs) to solve these types of eigenvalue problems. PINNs offer a data-free, physics-driven way to learn solutions of partial differential equations by incorporating the governing laws directly into the training process [1, 2]. We apply this method to compute multiple eigenpairs in both single- and multi-component reaction-diffusion systems, including scenarios that are especially difficult for conventional approachessuch as nonself-adjoint systems. To demonstrate the method, we focus on two benchmark models: the ZeldovichFrankKamenetsky (ZFK) equation [3], representing a single-component and self-adjoint case, and the FitzHughNagumo (FHN) system [4], which introduces a two-component, nonself-adjoint structure. Our PINN framework enforces critical physical properties like biorthonormality between left and right eigenfunctions and spectral ordering within the loss function itself. As a result, it produces eigenvalue and eigenfunction estimates that align well with results obtained through standard numerical simulations. This work highlights the potential of PINNs as a robust and adaptable approach to spectral analysis in complex reactiondiffusion systems.

***K*eywords** PINN · deep learning · reactiondiffusion · eigenvalue problems · FitzHughNagumo · ZeldovichFrankKamenetsky · adjoint

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