ABSTRACT BOOK

IV International Conference on Mathematics and Its Applications in Science and Engineering (ICMASE 2023)

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ICMASE 2023

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Preface

This abstract booklet includes the abstracts of the papers that have been presented at IV International Conference on Mathematics and its Applications in Science and Engineering (ICMASE 2023) which was held in U-tad (University Center for Technology and Digital Art), Spain between 12-14 July, 2023, via hybrid. The aim of this conference is to exchange ideas, discuss developments in mathematics, develop collaborations and interact with professionals and researchers from all over the world about some of the following interesting topics: Functional Analysis, Approximation Theory, Real Analysis, Complex Analysis, Harmonic and non-Harmonic Analysis, Applied Analysis, Numerical Analysis, Geometry, Topology and Algebra, Modern Methods in Summability and Approximation, Operator Theory, Fixed Point Theory and Applications, Sequence Spaces and Matrix Transformation, Modern Methods in Summability and Approximation, Spectral Theory and Diferantial Operators, Boundary Value Problems, Ordinary and Partial Differential Equations, Discontinuous Differential Equations, Convex Analysis and its Applications, Optimization and its Application, Mathematics Education, Applications on Variable Exponent Lebesgue Spaces, Applications on Differential Equations and Partial Differential Equations, Fourier Analysis, Wavelet and Harmonic Analysis Methods in Function Spaces, Applications on Computer Engineering, and Flow Dynamics. However, the talks are not restricted to these subjects.

Thanks to all committee members.

We wish everyone a fruitful conference and pleasant memories from ICMASE 2023.

Prof. Dr. Víctor GAYOSO MARTÍNEZ,

Prof. Dr. Fatih YILMAZ

Chairs, ICMASE 2023

IV International Conference on Mathematics and Its Applications in Science and Engineering (ICMASE 2023)

12-14 July 2023, U-tad (University Center for Technology and Digital Art)

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APPLICABILITY OF AI TO CRYPTOGRAPHIC ALGORITHMS

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ABSTRACT

Recently, the National Institute of Standards and Technology set CRYSTALS– Kyber as post-quantum public key encryption/key encapsulation mechanism standard, and CRYSTALS–Dilithium as post–quantum digital signature standard. These post quantum cryptosystems are also recommended for national security systems. As a result, it is important to identify and analyze the weaknesses and potential information leakage points, so that they can be resolved. In this talk, the newest side channel attacks based on artificial intelligence models against Kyber and Dilithium are presented, focusing on the specific function attacked.

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RECENT DEVELOPMENTS IN COMPUTING MATRIX FUNCTIONS WITH THE DOUBLE EXPONENTIAL FORMULAS

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ABSTRACT

Matrix fractional powers, matrix logarithms, and matrix exponentials are examples of matrix functions, which arise from a rich variety of applications such as fractional partial differential equations, control theory, lattice quantum chromodynamics, and quantum information. Among many approaches to computing matrix functions, we have focused on integral representations of matrix functions and solved them using numerical integration formulas. This talk will describe these recent developments in our numerical algorithms.

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NEW TRENDS ON MALWARE PROPAGATION: FROM IOT ENVIRONMENTS TO DRONE SWARMS

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ABSTRACT

Currently, due to the degree of digitalization of our society, the main goal of Mathematical Epidemiology is not only the development of mathematical models for the study of communicable diseases caused by biological agents but also the study and analysis of the propagation of malicious code (malware) on different types of networks. This new scenario has not been managed in a proper way since the study of the new malware models has been based on the same epidemiological framework than those devoted to biological agents. Then both the epidemiological coefficients and the types of incidences used in their development are defined analogously to those used in the case of biological agents. Consequently, the great majority of the proposed mathematical models to simulate malware propagation lack sufficient realism to consider them efficient. Likewise, new techniques related to Complex Network Analysis and Machine Learning that have appeared in recent years can be very effective in developing of these new families of models. The main goal of this talk is to show a reformulation the fundamentals of Mathematical Epidemiology to the case of the spread of malicious code and, at the same time, to explore the use of new techniques to design and analyze novel models that predict malware behavior in different scenarios such as IoT environments or drone swarms.

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CONTRIBUTED TALKS

SOME PROPERTIES OF ALGEBRAS OBTAINED BY THE CAYLEY-DICKSON PROCESS

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ABSTRACT

The Cayley-Dickson process gives us an alternative framework to view the construction of quaternions and octonions, being a method of obtaining new algebras of higher dimensions from the old ones. This implies an array operation on square arrays distinct from matrix multiplication.

By examining the Cayley-Dickson process is found a square representation for the octonions with a multiplication distinct from matrix multiplication. This representation also includes the one for complex numbers and the quaternions.

The Cayley-Dickson process for the real numbers is an iterative process that forms algebras over $\mathbb{R}$ with a conjugation involution. By applying it on the set $\mathbb{R}$, we obtain that $\mathbb{R}$ produces $\mathbb{C}$ then $\mathbb{H}$ then $\mathbb{O}$, taking the ordered, commutative, associative algebra $\mathbb{R}$ and gradually eliminating one adjective at a time, fact that illustrates the way how each of these algebras nests inside the next one. Using the previous construction of new algebras from the old ones, we try to explore their properties. Some of them already known are presented in this paper, but the question remains still open.

***K*eywords** Cayley-Dickson process · Quaternion division algebra · Octonion division algebra · Normed division algebra

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BLOCK-CENTERED COMPACT DIFFERENCE METHODS AND APPLICATION IN CONTAMINATION FLOWS

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ABSTRACT

Contamination flows in porous media are important in many applications, which are characterized by transport and nonlinear reaction processes in porous media in large-scale and long-term prediction and protection. In general, the reactions can be described as the kinetically controlled dissolution-precipitation reactions or the geochemical equilibrium reactions as hydrolysis aqueous complexation, oxidation-reduction, ion exchange, surface complexation, and gas dissolution-exsolution reactions. Mathematical models describing contamination flows in porous media are the time-dependent and coupled nonlinear partial differential equations. In this talk, we will first present our development of efficient block-centered domain-decomposition methods for solving contamination flows in parallel computing. We will then present block-centered compact differnece methods for solving the time-dependent partial differential equations and the nonlinear contaminant transport equations with adsorption. Meanwhile, the block-centered compact S-DDM scheme is also reported. The feature of the developed methods is that while keeping the advantages of the domain decomposition and the splitting technique, the S-DDM schemse preserve mass. Numerical experiments are given to show their performances. The developed algorithms work efficiently over multiple sub-domains over block-centered meshes, which can be applied in simulation of contamination flows.

***K*eywords** Block-Centered · Compact Difference · Contamination flow

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SOME PROPERTIES OF MATRICES OF QUATERNIONS

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ABSTRACT

In 1843, Irish mathematician William Rowan Hamilton made an important discovery in the field of algebra, namely quaternions. These are numbers with four components, which form a four-dimensional real algebra. Hamilton's discovery attracted the attention of the scientific world and paved the way for new discoveries in mathematics and physics.

Hamilton described all the properties of the set of quaternions, including the fact that the commutativity of multiplication of two quaternions is no longer fulfilled. This property posed a challenge for many mathematicians who were trying to work with quaternions, but at the same time, it offered a new perspective on algebra and opened the way for the development of new techniques and methods.

This paper presents some fundamental tools for developing matrix analysis over the real quaternion algebra. Specifically, it details the properties of matrices over real quaternions.

Quaternion algebra is an extension of complex numbers, and matrices over real quaternions are matrices with elements in the quaternion algebra. These matrices have interesting properties and can be used to solve problems in various fields, such as physics, engineering, or computer science.

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NUMERICAL SOLUTION OF AN ISENTROPIC EULER SYSTEM

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ABSTRACT

In this study, a special finite differences method for solving the following Cauchy problem of the simplified Euler system which models the flow of incompressible, irrotational and low viscosity fluid subject to a constant pressure

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{3}$$

is examined [1], [2]

The basic feature of the system of equations is that their solutions have jumps of unknown locations. These features present significant difficulties in implementing the classical methods to find the solution of the equation during their own time. For this goal, a special auxiliary problem which is the Bernoulli integral of the two-dimensional Euler's system of equations is suggested. The auxiliary problem permits us to develop effective and economical algorithms since the solution of it are smoother than the solution of the main problem [3], [4]. Finally, a finite difference method in the class of discontinuous functions is proposed to find the numerical solution of the nonlinear isentropic Euler system expressing the motion of polytropic gases.

Keywords 2D Euler's systems · Auxiliary problem · Finite differences schema in class of discontinuous functions

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A STOCHASTIC MAXIMUM PRINCIPLE APPROACH FOR A NASH EQUILIBRIUM OF A NONZERO-SUM GAME

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ABSTRACT

We develop a Stochastic Maximum Principle approach for two player constraint nonzero-sum stochastic differential game, which is modeled by Markov regime-switching jump-diffusion processes. We provide the relations between a usual stochastic control setting and a Lagrangian one. We present corresponding theorems for two different type of constraints, which lead us to find real valued and stochastic Lagrange multipliers, respectively. Then, we illustrate our results for an example of cooperation between a bank and an insurance company, which is a popular, well-known business agreement type, called Bancassurance. We investigate optimal dividend strategy for the company as a best response according to the optimal mean rate of return choice of a bank, and vice versa. We find out a Nash equilibrium for this game and solve the adjoint equations explicitly for each state.

***K*eywords** Stochastic Optimal Control · Stochastic Differential Games · Regime-switches · Insurance

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ON GENERALIZED LEONARDO NUMBERS AND FIBONACCI FUNDAMENTAL SYSTEM

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ABSTRACT

Recently, several families of sequences of numbers have been studied, such as Fibonacci, Pell, Lucas, Jacobsthal, Padovan and Perrin numbers, and their generalizations. One of these interesting sequences is the Leonardo numbers, $\{Le_n\}_{n\geq 0}$, defined by the recurrence relation

$$Le_n = Le_{n-1} + Le_{n-2} + 1,\ (n \geq 2), \tag{4}$$

with initial conditions $Le_0 = Le_1 = 1$. The Leonardo numbers was introduced by Catarino and Borges in [4] and continues to be explored in various works in the literature (see more in [1],[2], [3],[6], [7], and references there in). Since the recurrence relation (4) can be write under the form $Le_{n+1} = 2Le_n - Le_{n-2},\ (n \geq 2)$, we consider following generalization,

$$L_{n+1} = 2^d L_n - kL_{n-2},\ (n \geq 2), \tag{5}$$

with $d, k \geq 1$ and initial conditions $L_0 = \alpha_1, L_1 = \alpha_1$ and $L_2 = \alpha_2$, called generalized Leonardo numbers (5). Our goal is explore the generalized Leonardo numbers (5), through the properties of the Fibonacci fundamental systems related to the elements of this sequence, namely,

$$\begin{cases} L_{n+1}^{(j)} = 2^d L_n^{(j)} - kL_{n-2}^{(j)}, \text{ for } n \geq 3, \\ L_n^{(j)} = \delta_{n+1}^{(j)}; \text{ for } n = 0,\ 1,\ 2. \end{cases} \tag{6}$$

We describe explicitly the closed connection between the sequences $\{L_n^{(1)}\}_{n\geq 0}, \{L_n^{(2)}\}_{n\geq 0}$ and the fundamental sequences of generalized Leonardo numbers $\{L_n^{(3)}\}_{n\geq 0}$. Moreover, the matrix approach is considered for studying the combinatorial identities and the generalized Cassini identity for the generalized Leonardo numbers. In addition, the analytical aspect about each sequence of the generalized Leonardo numbers (5) is elaborated. Finally, the combinatorial and the analytical formula of the generalized Cassini identity are investigate.

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This approach has been applied for models of generalized sequences, as generalized Fibonacci numbers and the generalized Pell numbers (see more details in [5],[3] and [9]), and several results were established.

***K*eywords** Leonardo numbers · Fibonacci fundamental system · Combinatorial formulas · Analytical formulas · Cassini Identity

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A QUADRATIC ESTIMATION APPROACH FROM FADING MEASUREMENTS SUBJECT TO DECEPTION ATTACKS

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ABSTRACT

Over the last decades, the least-squares (LS) estimation problem of random signals from noisy measurements has been a hot research topic with interesting applications in many fields, such as engineering and economics, among others. The well-known Kalman filter is useful for providing the LS optimal signal estimator for linear systems subject to Gaussian and mutually independent initial signal and noise processes. This estimator is linear and, therefore, under ideal Gaussianity and independence conditions, the LS linear estimator is equal to the optimal one. However, in the presence of non-Gaussian disturbances, the LS linear estimator is, generally, non-optimal and computationally expensive, not to mention the case of complex systems with different uncertainties, where the LS linear filter experiences a significant performance degradation. Thus, other suboptimal estimators –such as polynomial and, in particular, quadratic estimators–, which strike a balance between estimation accuracy and computational complexity, have gained significant attention.

Although the LS quadratic estimation problem has been studied for different classes of linear and nonlinear systems where the signal evolution model is known (see, e.g., [1]-[3]), many practical situations where this model is not available may require other kind of information, such as covariance information. In this framework, based on the knowledge of the statistical moments –up to the fourth-order ones– of the stochastic processes involved in the observation equation, LS quadratic estimation algorithms have been proposed in different scenarios, such as measurements with correlated one-step random delays [4] and measurements with random packet dropouts [5]. However, to the best of the authors' knowledge, the quadratic estimation problem has not been studied in the class of systems with fading measurements exposed to stochastic deception attacks.

In this paper, using covariance information, the LS quadratic filtering and fixed-point smoothing problems are addressed under the assumption that the measurements are perturbed by both a multiplicative noise and a time-correlated additive

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noise, affected by the fading phenomena and exposed to random deception attacks. For the LS quadratic estimation approach, the signal and observation vectors are augmented by aggregation of the original vectors with their second-order Kronecker powers. Then, using Kronecker algebra rules and an innovation approach, the linear estimators of the original signal based on the augmented observations are obtained, providing the required quadratic estimators. A simulation example shows the feasibility of the proposed quadratic estimation algorithms; also, the superiority of the quadratic estimators over conventional linear ones is illustrated and the influence of the deception attack success probabilities on the estimation accuracy is analyzed.

***K*eywords** Least-squares quadratic estimation · Fading measurements · Multiplicative noise · Time-correlated additive noise · Deception attacks

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SOME IDENTITIES FOR BALANCING AND LUCAS-BALANCING NUMBERS IN BIDIMENSIONAL VERSION

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ABSTRACT

Actually, several sequences of numbers have been the research topic of many researchers. One example of such sequences are the balancing $\{B_n\}_{n\geq 0}$ and Lucas-balancing $\{C_n\}_{n\geq 0}$ numbers given by the recurrences relations

$$B_{n+2} = 6B_{n+1} - B_n,$$

with the initial conditions $B_0 = 0$ and $B_1 = 1$;

$$C_{n+2} = 6C_{n+1} - C_n,$$

with the initial conditions $C_0 = 1$ and $C_1 = 3$, respectively. (See for more information the works **[1]**, **[2]**, **[5]**, **[6]**, **[7]**, **[8]**, **[9]**, among others).
In this work we present a bidimensional extension of these two sequences, namely, bidimensional balancing numbers $\{B_{(n,m)}\}_{n,m\geq 0}$ and bidimensional Lucas-balancing numbers $\{C_{(n,m)}\}_{n,m\geq 0}$. These sequences are defined respectively as follows:

$$\begin{cases} B_{(n+1,m)} &= 6B_{(n,m)} - B_{(n-1,m)}, \\ B_{(n,m+1)} &= B_{(n,m)} - B_{(n,m-1)}, \end{cases}$$

with the following initial conditions $B_{(0,0)} = 0$, $B_{(1,0)} = 1$, $B_{(0,1)} = i$, $B_{(1,1)} = 1+i$, where $i^2 = -1$ and n and m are non-negative integers;

$$\begin{cases} C_{(n+1,m)} &= 6C_{(n,m)} - C_{(n-1,m)}, \\ C_{(n,m+1)} &= C_{(n,m)} - C_{(n,m-1)}, \end{cases}$$

with the following initial conditions $C_{(0,0)} = 1$, $C_{(1,0)} = 3$, $C_{(0,1)} = 1 + i$, $C_{(1,1)} = 3 + i$. (See the works **[3]** and **[4]** for more information).
About these new sequences, we study some identities satisfied by them involving sums of their terms.

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***K*eywords** Identities · Balancing numbers · Lucas-balancing numbers · Bidimensional version

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NUMERICAL ANALYSIS OF THE NONLOCAL ELLIPTIC PROBLEM WITH A p-KIRCHHOFF-TYPE TERM USING FEM

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ABSTRACT

There has been active ongoing research on the study of problems associated with the p-Laplace operator, which appears in a variety of physical fields [2, 3, 4]. In particular, a lot of attention has been devoted to nonlocal problems. One of the justifications of such models lies in the fact that in reality the measurements are not made pointwise but through some local average. This work is devoted to the study of the finite element method for a class of nonlocal elliptic problems associated with p-Kirchhoff-type operator. The convergence and *a priori* error estimates for the discrete formulation are established. Moreover, the finite element formulation is nonlinear, it can then be solved by Newton-Raphson's iterative but the main issue is that the Jacobian matrix of the Newton-Raphson method is full due to the presence of the nonlocal term thereby making computation expensive. The scheme presented here takes into account such issues. The predictions observed theoretically are validated by means of numerical experiments.

***K*eywords** Galerkin finite element method · Newton-Raphson method · Nonlocal diffusion term · p-Kirchhoff operator

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EXPONENTIATED WEIBULL MIXTURE CURE MODEL TO HANDLE CANCER DATA SETS

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ABSTRACT

It is generally assumed in survival data analysis that all study individuals will eventually experience the event of interest. However, it is to be assumed that a portion of these individuals will never be exposed to the event of interest. Typically, cure rate models are employed to model this type of data. We presented a maximum likelihood estimates analysis for the three-parameter exponentiated-Weibull (EW) distribution in the presence of cured individuals, censored data, and predictors in this study. A mixture cure model with EW distribution is presented in order to include a portion of unsusceptible (cured) individuals in the study. The proposed mixture cure model is the best model for fitting real-world data from cancer clinical trials.

***K*eywords** Mixture cure model · survival analysis · Exponentiated-Weibull distribution · maximum likelihood estimation

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MODELING OF NITROGEN, PHOSPHORUS, AND POTASSIUM CONCENTRATIONS IN LAKES AFFECTED BY SOIL FERTILIZATION

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ABSTRACT

The excessive use of fertilizer on agricultural land has led to a significant build-up of chemical nutrient stocks in the soil, which increases the risk of diffuse pollution of surface waters. The reservoirs to which these waters drain are also polluted by the chemical components of the fertilizer. This is a major environmental problem. Indeed, the fishery products contained in these water reservoirs are directly contaminated, and this remains a major public health problem. Thus, using ordinary partial derivatives, we established budget equations for the mass concentrations of the major components of fertilizer chemicals such as nitrogen (N), phosphorus (P) and potassium (K) found in each natural water reservoir. Using the Euler discretization method, we linearized these budget equations in order to formulate them numerically with AMPL and solve them. The numerical solutions obtained thus make it possible to predict the mass concentrations of the chemical elements (N, P, K) at the level of each water reservoir in time by taking the measures for a substantial reduction of the chemical content of fresh water.

***K*eywords** Farmland · Water Reservoirs · Nitrogen · Phosphorus · Potassium · Partial Differential Equations

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FIBONACCI FUNDAMENTAL SYSTEMS, MATRIX FORMULATION OF GENERALIZED FIBONACCI SEQUENCES AND APPLICATIONS TO SOME FAMILIES OF NUMBERS

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ABSTRACT

The purpose of this talk is to present the matrix formulation of the weighted generalized Fibonacci sequences and provide some of their properties. In this context, the fundamental Fibonacci system and the associated fundamental solution will play a crucial role, to prove interesting results, concerning analytical formulas, combinatorial formulas and other identities. The approach of generalized Fibonacci sequences, based on matrix theory, makes it possible to establish concrete mathematical applications related to number theory and other subjects. More precisely, we will illustrate the results obtained, by some concrete applications, related to special sequences of generalized Fibonacci numbers and generalized Pell numbers, as well as to other types of sequences of known numbers. Finally, concluding remarks and observations are given.

***K*eywords** Matrix formulation of generalized Fibonacci sequences · Generalized Fibonacci Fundamental system · Generalized Fibonacci-Pell numbers and Cassini identities

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A MATHEMATICAL MODEL UNIFYING WETTING AND SOLIDIFICATION PHENOMENA

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ABSTRACT

The spreading of a liquid over a solid surface and the propagation of a solidification front through an initially warm liquid, previously studied as unrelated phenomena, have been brought together by the arrival of additive manufacturing where liquid drops impacting and solidifying on top of each other form complex 3-dimensional structures. The first of the above phenomana, known as 'dynamic wetting', has been the subject of research for 50 years with the focus primarily on finding the relationship between the contact angle formed by the free surface with the solid and the speed of the three-phase contact line. However, as recently found experimentally [1], no such relationship exists in principle, which largely devalued previous research effort [2]. On the other hand, the studies of the propagation of solidification fronts deal almost exclusively with the Stefan regime and its extensions towards kinetic undercooling, and they are irrelevant to the situation where the liquid is initially warm and the solidification front propagates not only in the normal direction but also along the solid substrate. Thus, the complex dynamics of both phenomena and their interaction make it necessary to develop a mathematical model which would (i) describe dynamic wetting on its own without imposing any angle-versus-speed relationship, (ii) describe solidification in the pre-Stefan regime and (iii) consider both dynamic wetting and solidification in the same conceptual framework. The present work addresses this challenge and shows that both dynamic wetting and solidification are actually particular cases in a general class of flows where interfaces form and/or disappear displaying dynamic properties in the process. In dynamic wetting, a freshly wet solid surface, i.e. a new liquid-solid interface, is created as the liquid spreads over the solid whilst solidification is essentially a process where continuous formation of the liquid-solid interface, as the liquid is taken into the solidification zone, is coupled with its disappearance, as the solidification zone loses its material to the solid phase. A particular situation illustrating the model's properties is the onset of solidification where it starts from pure interface formation and then evolves through non-equilibrium solidification towards the Stefan regime.

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***K*eywords** Dynamic wetting · Solidification · Interface formation · Structureless approach · Non-equilibrium thermodynamics · Pre-Stefan regime

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ELLIPTIC BIQUATERNIONIC SEQUENCE WITH VIETORIS' NUMBERS AS ITS COMPONENTS

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ABSTRACT

Special integers sequences have been the center of attention for many researchers, in this work we will focus our attention to a sequence of rational numbers denoted by *Vietoris sequence*, $\{v_n\}_{n\geqslant 0}$, which is defined by

$$v_n = \frac{1}{2^n}\binom{n}{\lfloor \frac{n}{2} \rfloor}, \qquad n \geqslant 0,$$

where $\lfloor \cdot \rfloor$ is the floor function. The first Vietoris' numbers of the sequence are

$$1, \frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{3}{8}, \frac{5}{16}, \frac{5}{16}, \frac{35}{128}, \frac{35}{128}, \frac{63}{256}, \frac{63}{256}, \frac{231}{1024}, \frac{231}{1024}, \cdots,$$

which is related with the sequence A283208 in the OEIS in [5]. It is also known that the recurrence relation for this sequence is given by the following identity:

$$v_n = \begin{cases} \dfrac{n}{n+1} v_{n-1}, & n \text{ odd} \\ v_{n-1}, & n \text{ even} \\ 1, & n = 1 \end{cases}.$$

In 1958, L. Vietoris published a result in [7] where this sequence appeared. This sequence has applications in harmonic analysis, as can be seen in the work of R. Askey and J. Steinig, in [1], as well as, in the theory of stable holomorphic functions presented in the paper [6] by St. Ruscheweyh and L. Salinas.

Some properties of Vietoris' number sequence and some techniques, using special types of matrices that generates this number sequence were presented in [3] by P. Catarino and R. de Almeida.

In [2], we study some properties of the quaternionic sequence with Vietoris' numbers as its components, $\{Q_n\}_{n\geqslant 0}$,

$$Q_n = v_n + v_{n+1}\mathbf{i} + v_{n+2}\mathbf{j} + v_{n+3}\mathbf{k}, \quad n \in \mathbb{N}, \tag{7}$$

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where $\{1, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$ is the standard basis in $\mathbb{R}^4$ satisfying the following multiplication rules:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1, \quad \mathbf{ij} = -\mathbf{ji} = \mathbf{k}, \quad \mathbf{jk} = -\mathbf{kj} = \mathbf{i}, \quad \mathbf{ki} = -\mathbf{ik} = \mathbf{j}.$$

The system of elliptical numbers is given by

$$\mathbb{C}_p = \{x + Iy : x, y \in \mathbb{R}, \quad I^2 = p, \quad p < 0\},$$

some properties of these type of numbers can be seen in [4]. Using the elliptic numbers, the set of elliptic biquaternion is defined as

$$\mathbb{H}_{\mathbb{C}_p} = \{A_0 + A_1\ \mathbf{i} + A_2\ \mathbf{j} + A_3\ \mathbf{k}, \quad A_s \in \mathbb{C}_p,\ s = 0, 1, 2, 3\}\,.$$

In this paper, using (7), we introduce an elliptic biquaternionic sequence, $\{V_n^p\}_{n \geqslant 0}$, given by

$$V_n^p = Q_n + IQ_{n+1}, \quad I^2 = p, \quad p \in \mathbb{R}^+, \quad n \in \mathbb{N},$$

and discusse some of its properties.

***K*eywords** Biquaternion · Elliptic numbers · Vietoris' number

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SEQUENCES OF UNCOUNTABLE ITERATED FUNCTION SYSTEMS. THE CONVERGENCE OF THE SEQUENCES OF FRACTALS AND FRACTAL MEASURES ASSOCIATED.

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ABSTRACT

In this paper, we consider a sequence of uncountable iterated function system (U.I.F.S.). Each term of this sequence is built using an uncountable family of contractions and a linear and continuous operator. For each U.I.F.S. of the sequence we have an associated attractor, a Markov-type operator and a fractal measure.
First, we consider the Markov operator on the Borel normalized measures and thus, we obtain the Hutchinson (scalar) measure (or the fractal (scalar) measure). We prove that the support of the fractal measure is the attractor of the U.I.F.S.
The second step is to consider fractal vector measures. For this aim, we construct the Markov-type operator, using the family of contractions and an uncountable family of linear and continuous operators. Then, we consider a new operator, which adds a fixed measure μ° to the Markov operator. The fixed point of this new operator is called *the fractal vector measure*. The measure μ° can be chosen such that the support of the fractal vector measure is, also, the attractor of the U.I.F.S.
The third step is to consider a sequence of U.I.F.S., using a sequence $(T_n)_n$ of linear and continuous operators and an uncountable family of contractions, $(\omega_\alpha)_\alpha$. Then $[(T_n \circ \omega_\alpha)_\alpha]_n$ is a sequence of U.I.F.S. We suppose that $T_n \overset{\|\cdot\|}{\to} T$, T being, also a linear and continuous operator. We denote: K_n and μ_n^* the attractor and the fractal vector measure associated to $(T_n \circ \omega_\alpha)_\alpha$, and K, respectively μ^* the attractor and the fractal vector measure associated to $(T \circ \omega_\alpha)_\alpha$. The problem that we solve is: it is true that $K_n \to K$ and $\mu_n^* \to \mu$, in some appropriate metrics?
Finally, we give some examples, to illustrate the theoretical concepts.

***K*eywords** Iterated function system · attractor · fractal measure · Markov-type operator · vector measure

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ON GENERALIZED COMPLEX SPACE FORMS SATISFYING CERTAIN CONDITIONS ON THE HOLOMORPHIC CURVATURE TENSOR

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ABSTRACT

In 1989, Z. Olszak has worked on the existence of a generalized complex space form. In 1998, M. Prvanović has introduced a tensor of Kaehler type for an almost Hermitian manifold, this tensor called holomorphic curvature tensor and reduces to the Riemannian curvature tensor R in an almost Kaehler manifold. Then M. Prvanović gave some properties about such tensor.

In this work, first the holomorphic curvature tensor of generalized complex space forms are presented. Moreover, we investigate generalized complex space forms satisfying some conditions on the holomorphic curvature tensor. It is proved that the generalized complex space form $M(f_1, f_2)$ of dimensional $(m > 2)$ satisfying $HR = 0$ is Einstein.

Finally, Walker type identities for holomorphic curvature tensor of generalized complex space forms are studied. We show that an m-dimensional $(m > 2)$ generalized complex space form satisfies Walker type identities.

***K*eywords** Generalized complex space form · Holomorphic curvature tensor · Pseudosymmetric manifold · Tachibana tensor · Walker type identity

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THE GENERAL SYSTEM OF GERDJIKOV–IVANOV EQUATION

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ABSTRACT

Our goal in this study is to present solutions to the Gerdjikov-Ivanov system by developing the Marchenko method for a first-order system of two linear ordinary differential equations. We construct the potentials and the scattering solutions to the linear system from the solution to the Marchenko system. Such a linear system arises in the solution to a pair of integrable nonlinear partial differential equations known as the derivative nonlinear Schrödinger equations via the so-called inverse scattering transform method.

***K*eywords** Marchenko method · General derivative NLS system · Inverse scattering transform · Gerdjikov-Ivanov system

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ON MIN MATRICES INVOLVING CHEBYSHEV POLYNOMIALS

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ABSTRACT

In the present study, by inspiring the studies in literature, we construct one type of Min matrix from symmetric family whose entries are Chebyshev polynomials of the first kind. Afterwards, we investigate some linear algebraic properties and obtain inequalities by using matrix norms. Furthermore, we obtain some summation identities for Chebyshev polynomials. In addition to all these, we shed light on the results we obtained to boost the clarity of our paper with the illustrative examples.

***K*eywords** Min matrices · Chebyshev polynomials · Matrix norm · Determinant · Inverse

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ON CIRCULANT MATRICES INVOLVING FIBONACCI POLYNOMIALS

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ABSTRACT

In this paper, we consider circulant matrices associated with Fibonacci polynomials, i.e., $C_{F_n} = Circ(F_1(x), F_2(x), F_3(x), ..., F_{n-1}(x))$. Then, we compute its determinant by following two different ways. For this purpose, we benefit from the Chebyshev polynomials of the second kind and exploit some matrix operations. Moreover, we obtain Euclidean norm, upper and lower bounds for $\|C_{F_n}\|_2$. Finally, we illustrate our result in tables and give a graph of C_{F_n}.

***K*eywords** Fibonacci polynomials · Determinants · Circulant matrices

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APPROXIMATION BY NEW MODIFICATION OF BERNSTEIN OPERATORS

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ABSTRACT

In this presentation, we investigate a new modification of Bernstein-Kantorovich operators. We obtain the rate of convergence by means of the modulus of continuity. We demonstrate the Voronovskaya-type theorem for the newly constructed operator. We represent some illustrative graphics to show the convergence of the constructed operators and we give some numerical results.

***K*eywords** Bernstein-Kantorovich operators · Modulus of continuity · Voronovskaya-type theorem

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A Generalization of Szász-Kantorovich Operators by Using Special Polynomials

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Abstract

In this talk, a new generalization of Szász-Kantorovich operators involving Appell polynomials is introduced. The convergence theorem of the newly constructed operators is obtained. Moreover, some approximation results are studied. Also, approximation results of Gould-Hopper polynomials and Hermite polynomials are given. Finally, Voronovskaya-type theorem is presented.

***K*eywords** Appell Polynomials · Gould-Hopper Polynomials · Hermite Polynomials

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SERVICE-LEARNING ACTIVITY IN STATISTICS COURSE

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ABSTRACT

Learning through service, a combination of community-based activities with learning objectives, or experiential education, are some features of service-learning (SL) methodology, which improves the realities where the service is performed, and which considers who receives the service as a central element [5]. The difference between voluntary and SL is that this is developed while learning. Several courses implement this approach as part of its methodology, this is the case of Engineering Design [1], Mechanical Engineering [3], Economics [2], Medicine [6], Statistic [4], etc. A SL activity was proposed to undergraduate sophomore students from Industrial Engineering degrees. Students developed a teamwork as part of their statistics course collaborating with a local entity. To make this possible, teachers identified a community need in line with the course contents and after that students were invited to participate. A teamwork was proposed to all students and one group decided to collaborate with an association that helps families in need. The work consisted of creating a database with the data gathered by volunteers and after that present a basic descriptive analysis. The built database and the analysis performed were grounded on the association needs to better help the families that it serves. This SL activity was assessed as part of the course activities.

***K*eywords** Service-Learning · Statistic · Industrial Engineering

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METHOD OF HYDRODYNAMIC IMAGES AND QUANTUM CALCULUS IN FOCK-BARGMANN REPRESENTATION OF QUANTUM STATES

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ABSTRACT

The hydrodynamical analogy is a fruitful approach in many fields of physical science and mathematics, including electromagnetic theory, quantum mechanics, nuclear physics, dynamical systems and complex analysis. In quantum mechanics, the hydrodynamic representation of wave function by Madelung gives intuitive picture of probability flow, propagating in quantum fluid. In complex analysis, hydrodynamic interpretation gives meaning to an analytic function as descriptive of incompressible and irrotational fluid in plane and classification of singularities of the function by point vortices, sinks and sources, etc. Here we propose a new approach to quantum states in Fock space in terms of classical hydrodynamics. By Madelung type conformal mapping of entire complex analytic function, representing the a wave function of quantum states in Fock-Bargmann representation, we define the complex potential, describing these quantum states by incompressible and irrotational classical hydrodynamic flow. In our approach, zeros of the wave function appear as a set of point vortices in plane with the same strength. This allows us to interpret these vortices as images of the one real vortex in a bounded domain, with vanishing normal velocity across the boundary curve. For simple domains like the wedge, the strip and the annular domain, the distribution of the images is determined by the wedge theorem, the strip theorem and the two circle theorem, respectively. For the strip domain, the infinite set of periodic images is described by elliptic functions. For the wedge and the annular domain it can be represented naturally by the q-periodic functions of the quantum or the q-calculus, where the stretching from the origin and the rotation, result from inversion in two circles and reflection in two lines, correspondingly. For the wedge domain, when complex number q is a root of unity, $q^{2n} = 1$, the kaleidoscope of finite number of images is located at vertices of regular polygon and possess the quantum group symmetry. For the annular domain with real q, the infinite set of images is described by Jackson q-exponential functions. We show that these functions represent

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the wave functions of quantum coherent states of the q-deformed quantum oscillator in q-Fock-Bargmann representation and describe the infinite set of point vortices, distributed in geometric progression. For the so called displaced coherent states, created by adding n quanta to the coherent states, we obtain the set of point vortices in the displaced positions. The corresponding kaleidoscope of vortex images in Fock-Barmann representation of displaced quantum states and questions of separability and entanglement of the states are discussed. This work was supported by BAP project 2022IYTE-1-0002.

***K*eywords** quantum calculus · Fock-Bargmann representation · vortex images · coherent states

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ALGEBRAIC AND QUANTUM MECHANICAL APPROACH TO SPINORS

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ABSTRACT

Clifford algebras and quantum mechanics are closely associated to each other. We are familiar with some popular terms like algebra, light polarization, quantum spin etc. Dirac spinors, Majorana spinors and Weyl spinors as subspaces of Clifford algebras with some remarkable algebraic features are to be discussed in particular. Furthermore, we are interested to exhibit how quantum spin state and classical polarization of light waves can be derived from one another along with the Bloch sphere and the Poincare sphere representations.

Keywords Bispinors · $\mathcal{C}l_2$ and $\mathcal{C}l_3$ algebras · Lie algebras · Lie groups · Lorentz transformation

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A NUMERICAL APPROACH FOR A SINGULARLY PERTURBED MIXED TYPE DIFFERENTIAL-DIFFERENCE EQUATION

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ABSTRACT

In this paper, we deal with the singularly perturbed boundary value problem (SPBVP) for a class of first-order differential-difference equations with the mixed type (delay as well as advance). These problems appear in science and engineering fields such as the study of human pupil light reflex, first-exit problems in neurobiology, models of physiological processes and diseases, optimal control theory, models of climate systems, optically bistable devices and signal transmission, quantum photonic systems ([1, 2] and reference therein). Especially, model of nerve excitation and propagation of electronic impulses, nerves has been described these equations, which are usually known as the FitzHugh–Nagumo equations, which are considered as electric cables, through which electric current flows (for more details see [3]).

On the other hand, for small values of perturbation parameter ε, standard numerical methods for solving SPBVPs are unstable and do not give accurate results. Therefore, it is important to develop suitable numerical methods for solving these problems, whose accuracy does not depend on the parameter value ε, i.e., methods that are convergent ε-uniformly. Some of these are adapted meshes such as Shishkin mesh, and Bakhvalov mesh used together with the finite difference method [4].

We obtain the numerical solution to the problem under consideration by using the finite difference method for the adapted mesh. Firstly, we present some important properties of the exact solution of the problem, which are necessary for error analysis of the method. Next, in order to the numerical solution of this problem, we use a fitted difference scheme on a piecewise uniform mesh of Shishkin type which is accomplished by the method of integral identities with the interpolating quadrature rules with remainder term in integral form. It has shown that it gives almost first-order uniform convergence in the discrete maximum norm, independently of the perturbation parameter. Finally, we present the numerical experiments that their results support of the theory.

Keywords Singular perturbation · Differential-difference equation · Uniform convergence

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ON A NEW TYPE DUAL HYPERBOLIC NUMBERS

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ABSTRACT

Fibonacci numbers and the golden ratio associated with these numbers have been the focus of attention among mathematicians for many years. The Fibonacci numbers is a sequence of numbers which is given also by the second order linear recurrence relation. So are the Fibonacci-Lucas numbers. Over the years, new number sequences have also been defined and examined their several properties. One of them is the generalized John number sequences defined by Soykan last year. This sequence, being related to Pell sequence, can also given by the third order linear recurrence relation and its special cases when the initial values fixed are John and John-Lucas numbers. On the other hand, hypercomplex number systems extends the real numbers, e.g. complex numbers, hyperbolic numbers and dual numbers. These number systems are in two dimensions and commutative. Hyperbolic numbers with complex coefficients being introduced in the first half of 19th century are in 4 dimensions. In the recent past, the commutative dual numbers with complex coefficients were introduced. Similarly, hyperbolic numbers with dual coefficients were introduced by Akar, Yüce and Şahin and they called these numbers with dual hyperbolic numbers. The dual hyperbolic numbers form a commutative ring, real vector space and an algebra. After this number system defined, Fibonacci, Pell, Jacobsthal, etc. numbers were introduced in dual hyperbolic sense by taking the real, the dual, the hyperbolic and the dual hyperbolic part of the dual hyperbolic numbers being taken as the terms of Fibonacci, Pell, Jacobsthal, etc. in a certain rule, respectively. However, there has been no work about the generalization of John numbers in dual hyperbolic sense. In this paper, we introduce a new type dual hyperbolic sequence of numbers, which will be called by dual hyperbolic generalized John numbers. As special cases, we deal with dual hyperbolic John and dual hyperbolic John-Lucas numbers. We present several important properties of this new sequence, such as Binet's formulas, generating functions and the summation formulas. Moreover, we give special identities and matrices related with these sequences.

Keywords Dual Hyperbolic Generalized John Numbers · Dual Hyperbolic John Numbers · Dual Hyperbolic John-Lucas Numbers

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A FINITE DIFFERENCE METHOD FOR COUPLED SYSTEM OF SINGULARLY PERTURBED DIFFERENTIAL EQUATIONS

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ABSTRACT

System of differential equations occur in many scientific fields such as biology, chemistry, etc. Traditionally, these models occur in the mathematical formulations of the various problems such as deep neural network, electric circuits, feedback control systems, iodate-arsenous, acid reactions, cell growth, etc [1]. A class of these sytems are systems of singularly perturbed differential equation. These equations play a significant role in modeling of applied sciences. For instance Henri's enzyme–substrate reaction scheme can be modelled as a system of singularly perturbed problem [3].

In this study we deal with a initial value problem for system of partially singularly perturbed equations. These type systems contain small parameter (ε) which multiplying by the first derivative of one of the unknown functions. Unknown function multiplied by small parameter is called fast variable, the other function is slow variable. The solution of singularly perturbed problem varies rapidly where it has initial layer, meanwhile the solution of unperturbed problem changes more slowly [4].

In such problems, classical methods can not be applied due to the existence of boundary (or initial) layer. Therefore, adapted meshs gain more importance at this juncture for numerical solutions. Robust numerical techniques have been developed for singularly perturbed problems, but for system of equations a few results are reported in the literature, see [2]. For the past two decades, studies on the numerical solution of systems of singularly perturbed differential equation are quite remarkable.

We establish a difference scheme on Bakhvalov mesh and Shishkin mesh with the remainder terms in integral form. We estimate error and stability anaylsis in both meshs and prove that the numerical approaches generated by the method presented are essentially first order convergent in the maximum norm, uniformly with respect to the $\varepsilon-$perturbation parameter. Finally we present an example for illustrate the theoretical results provided. The computational results for presented method are displayed in tables and graphics.

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***K*eywords** Singular perturbation · System of differential equation · Finite difference method · Uniform convergence

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FEKETE-SZEGÖ PROBLEM FOR CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS RELATED TO THE COMBINATION OF DIFFERENTIAL OPERATORS

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ABSTRACT

In this study, we introduce and deal with some new subclasses of analytic functions defined by the combination of Deniz-Özkan and Ruscheweyh differential operators, and obtain coefficient estimates and Fekete–Szegö inequalities for these new subclasses.

***K*eywords** Analytic functions · Univalent functions · Fekete-Szegö problem · Starlike and convex function of complex order · Deniz-Özkan and Ruscheweyh differential operator

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STATISTICAL MODELING OF AVERAGE NUSSELT NUMBER

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ABSTRACT

In this study, an important heat transfer and fluid flow parameter is statistically modeled by using the data obtained from a numerical process. The two dimensional (2D), time dependent dimensionless equations of natural convection flow is numerically solved by global radial basis function (RBF) method. A trained data and a distinct test data, in which dimensionless numbers Rayleigh and Prandtl numbers are inputs and average Nusselt number is the output, are generated by saving the numerical results. Then, average Nusselt number is modelled by Trilayer Neural Network (TNN). A 2D curve fitting idea in view of interpolation is also examined. In terms of mean squared error metric, TNN gives the best goodness of fit results comparing to curve fitting. On the other side, it is shown that curve fitting is also an alternative for modelling. This modeling issue enables one to get the desired result without making heavy numerical calculations many times.

***K*eywords** Trilayer Neural Networks · global RBF · average Nusselt number · curve fitting

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SOLO TAXONOMY IN THE EVALUATION OF ENGINEERING STUDENTS: A CASE STUDY IN MATHEMATICS.

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ABSTRACT

With the development of skills, considered essential for the academic and professional path, the teacher seeks to discover how to assess these skills in an authentic way. Student assessment is one of the key points of their learning process. It is important that teachers use methodological tools, such as the SOLO taxonomy (Structure of Observed Learning Outcomes), to assess their students' learning [1, 2]. This study intends to be another contribution to mirror and think about assessment as an integral part of the teaching process and, above all, of learning, due to its important role in the academic success of students and the significant impact it has displayed in the improvement of learning. The main purpose of this study is to incite about the evaluation and in what way it can influence the teaching-learning process in the school environment. This is a qualitative investigation, which takes the SOLO taxonomy as an approach, through the analysis of evaluations carried out during the last 5 years in online and face-to-face tests of Mathematical Analysis of the 1st year of the Degree in Electrical and Computer Engineering. All the questions asked in the exams were categorized according to the type of knowledge the student should have in each answer. The results indicate that there are topics of the syllabus that are more frequent than others and that the complexity of the exam-type questions is different, with greater evidence between online and face-to-face exams. The grades accredited in each curricular topic are distributed according to their importance and the type of exam, with the SOLO index of the online exams, being higher than that of the face-to-face exams. In conclusion, this study reinforces the idea that assessment, when well operated and understood, works as a basis and efficiently contributes to the student learning process.

***K*eywords** SOLO Taxonomy · skills · assessment · teaching and learning

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IS COLLABORATIVE LEARNING A VOLUNTARY PROCESS?

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ABSTRACT

The process of learning Mathematics in Engineering courses has been extensively studied and investigated, with the use of Information and communication technologies (ICT) tools and digital platforms becoming increasingly prevalent in recent years. The years 2020 and 2021 were difficult years for students who were already attending higher education but also for those who were in the final years of secondary education preparing themselves to enter higher education. As a result of the pandemic, the teaching processes were adapted, and the assessments were the possible ones in the context that we all live and experienced. Thus, students arriving at higher education, and even those already there, need innovative and stimulating teaching and learning practices that quickly motivate and involve them in the teaching/learning processes. At the same time, ICT tools and digital platforms have seen their indiscriminate use in the last two years, not without at times, teachers and even students questioning whether they were being used in the best possible way and whether they were being taken full advantage of. Simultaneously, face-to-face group work and involvement with the needs of colleagues lost some space for achievement and effectiveness. The preference, assumed or disguised, for individual work and the visible reduction in solidarity with colleagues with greater difficulties, except probably in niches of friendship that come from previous school groups, was an issue/question that was posed at the beginning of this study. However, the COVID-19 pandemic has highlighted the need for innovative and stimulating teaching and learning practices that motivate and involve students in the learning process. Collaborative learning (CL) has been proposed as a solution, with the Padlet platform (https://padlet.com/) being a popular tool for facilitating such interactions. In this paper, the authors propose two collaborative learning platforms, one for students of Statistical Methods in Informatics Engineering and other for Mathematical Analysis I, with the aim of creating a space for virtual interaction among students. The study enrols 533 students from Informatics Engineering, many of whom are student workers, and from Mechanical Engineering 20 students,

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who were divided into small groups to foster teamwork and cooperation. The students are encouraged to investigate the topics taught in class, using bibliographical references and creative resources to deepen their understanding of the subject matter. The Padlet platform is used to facilitate collaboration and engagement ([1], [2], [3]), allowing students to share their resolutions of proposed exercises, correct or comment on colleagues' work, and access all work developed by their peers. In the case of Informatics Engineering graduation, the professors corrected the students' resolutions regularly, evaluating their involvement in the learning process and their collaboration and solidarity with colleagues. The paper discusses the results of this experiment, including the level of student participation, their collaboration with colleagues, and their overall performance in the course, highlighting the evidence of when it is, or it is not fruitful to use this type of activities, and which are their probable causes.

***K*eywords** Collaborative Learning · Active Methodologies · Padlet · Mathematics · Solidarity

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ON UNIVALENCE CRITERIA FOR ANALYTIC FUNCTIONS DEFINED BY A LINEAR MULTIPLIER DIFFERENTIAL OPERATOR

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ABSTRACT

In this paper we obtain sufficient conditions for univalence of analytic functions defined by a linear multiplier differential operator. Also, we give some results for special cases of parameters.

***K*eywords** Linear multiplier differential operator · Analytic functions · Univalent functions

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APPLICATION OF DISCRETE WAVELET TRANSFORM AND TREE-BASED ENSEMBLE MACHINE LEARNING FOR MODELING OF PARTICULATE MATTER CONCENTRATION

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ABSTRACT

The study of air pollution is an extremely important and urgent problem to be solved on a global and local scale. In this field, huge arrays of measurement data are accumulating, for the analysis of which various approaches based on mathematical, statistical, and machine learning (ML) methods are developed. In this paper, we investigate the application of different discrete wavelet transforms (DWT) families, coupled with state-of-the-art ML algorithms to predict concentrations of particulate matter PM10. Average daily data for this pollutant and several meteorological time series for a period of 630 days were used. A hybrid type models with wavelet decomposition of the initial time series and the application of predictive ensembles, were obtained. All models are cross-validated. The influence of the predictor meteorological time series was established. The models are applied for short-term pollution forecasts.

***K*eywords** Time series · Particulate matter · Wavelet transform · Ensemble tree learning · Hybrid model

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ON STRONG FUZZY PARTIAL METRIC SPACES

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ABSTRACT

In the last decades, the studies of fuzzy metrics and their extensions have attracted the attention of many authors not only studying theoretical such as topological properties but also giving different applications to engineering problems. As an extension to the fuzzy setting of the concept of partial metric space, in literature, there are different fuzzy partial metric space approaches. One of them has been given by Gregori et al.[2], in 2019, which was introduced by giving the axiom of triangle inequality based on the residuum operators. This structure has been a more appropriate approach axiomatically as an extension of both the partial metric and fuzzy metric spaces when compared with the others. However, since it is not easy to find different examples when one works with the residuum operators, in the recent papers, we face to lack of examples of fuzzy partial metrics. Motivated by this fact, in this work, we aim to generate different fuzzy partial metrics from partial metrics and vice versa. For this aim, we first introduce the notion of strong fuzzy partial metric spaces due to establish a connection between the mentioned structures. Then we investigate some properties of strong fuzzy partial metric spaces and provide different examples by considering distinct t-norms. In continuation of this work, we give some techniques to construct strong fuzzy partial metrics from existing partial metrics by means of pseudo-inverse of the additive generator of continuous Archimedean t-norms. Besides, we take into consideration of the minimum t-norm which is continuous but non-Archimedean to obtain a similar construction with the same technique. We also show that strong fuzzy partial metrics can induce partial metrics via residuum operators. Moreover, we study whether the topology of the partial metric obtained coincides with the topology of the fuzzy partial metric from which constructs or not and vice versa.

***K*eywords** partial metric · fuzzy partial metric · Archimedean t-norm · pseudo-inverse function · residuum operator

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TEACHING MATHEMATICS IN STEM EDUCATION.

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ABSTRACT

Teaching mathematics in STEM education is a vital component of developing students' analytical thinking, problem-solving skills, and logical reasoning abilities. To effectively teach mathematics in STEM, educators should employ strategies such as active learning, integration of technology, problem-based learning, differentiated instruction, cross-disciplinary connections, visualization, real-world applications, collaborative learning, formative assessment, and cultivating a growth mindset. The integration of technology plays a significant role in enhancing mathematics instruction. Educators can utilize interactive software, graphing calculators, spreadsheets, online resources, virtual manipulatives, coding and programming, online collaborative platforms, data visualization tools, virtual reality, augmented reality, online tutorials, and videos to engage students and deepen their understanding of mathematical concepts. By incorporating technology, teachers can provide dynamic visualizations, simulations, and interactive activities that enable students to explore multiple representations of mathematical ideas. Technology tools also support data analysis, graphing, problem-solving, and collaborative learning experiences. Furthermore, technology facilitates real-world connections and applications of mathematics, helping students recognize its relevance in various STEM fields. Overall, the integration of technology in mathematics instruction empowers students to actively participate in their learning, visualize abstract concepts, solve complex problems, and develop essential mathematical and STEM skills. By incorporating these strategies, educators can create a dynamic and engaging mathematics learning environment that prepares students for future STEM pursuits.

***K*eywords** Mathematics education · STEM education · Teaching strategies · Active learning · Critical thinking

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AN APPLICATION OF LINEAR DIOPHANTINE FUZZY SETS TO THE EDGE DETECTION TECHNIQUES

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ABSTRACT

The utilization of fuzzy logic within the domain of digital image processing offers a lot of advantages such as encompassing the management of uncertainty, adaptability to variations, noise tolerance, adaptive classification, integration of expert knowledge, handling of ambiguous or intricate categories, and seamless integration compared to the other techniques. These advantages support enhanced precision and adaptability in the realm of image processing, empowering the meticulous and versatile manipulation of visual data. The process of edge detection, an essential cornerstone in the field of digital image processing, performs a pivotal role in the segmentation of foreground objects from the image background. So, it facilitates subsequent analysis and comprehension of the image's underlying structural properties through complex computational procedures. This complex process can be handled with fuzzy sets, where the degree of an element belonging to the set is represented by using membership functions. In the step of edge detection for the effective analysis and processing of visual data, it becomes necessary to introduce the generalization of fuzzy sets into image processing techniques when the fuzzy sets are not enough to process the data. The notion of Linear Diophantine fuzzy sets is a generalization of fuzzy sets where the use of reference parameters corresponds to membership and non-membership grades. The aim of this study is to give an application of linear Diophantine fuzzy sets to edge detection of images. For this aim, we conduct a comprehensive evaluation to ascertain the similarity values using the linear Diophantine fuzzy similarity measure by leveraging the gray normalized membership values associated with fundamental edge detection techniques. We also investigate whether there are specific parameter values at which these techniques exhibited resemblances to one another, thus we understand more accurately their interrelationships and potential applications. In the mentioned process, we obtain the results by using MATLAB which has the advantages of a powerful data processing structure.

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***K*eywords** Image processing · Edge Detection · Linear Diophantine Fuzzy Sets · Similarity

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ON SOME FIXED POINT THEOREMS

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ABSTRACT

Metric spaces are an important tool in the transition from classical analysis to modern analysis. Fixed point theory in metric spaces started with Banach contraction principle in 1922. According to this principle "(X, d) is a complete metric space and $f : X \to X$ is a self mapping. If there is a constant $0 \leq k < 1$ such that the inequality $d(fx, fy) \leq k.d(x, y)$ is provided for each for each $x, y \in X$, then f has unique fixed point".

In recent years, fixed point theory appeared as a fundamental subject in nonlinear analysis. It has useful application in Mathematics as well as other sciences such as computer science, engineering, economy and physics. In the literature, there are many generalisations of fixed point theorems in both metric and generalized metric spaces because of their usefulness. One of the generalized metric spaces defined in recently is F-metric spaces which have properties of nonnegativity, symmetry and a special form of triangular inequality.

In this study, a rational $\alpha-$ admissible type contraction principle will be defined in F-metric spaces as a generalisation of Banach contraction and fixed point theorems will be proved for this type contraction.

Keywords F-metric · fixed point · contraction principle

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AN ALGORITHM FOCUSED ON DETERMINING THE BEST PARAMETERIZATION TOOL FOR UNCERTAIN ENVIRONMENTS BASED ON DECISION MAKING

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ABSTRACT

Decision-making problem in an uncertain environment is found prime importance in current periods of time. Innovative methods based on soft set theory with applications in many fields of the daily life of uncertain environments have already been developed and proposed. In this paper, we find out the best possible parameter from the given fixed set of parameters in any soft sets over the universe U, which is given for the solution of an uncertainty problem. Moreover, we construct an algorithm, which proceeds toward an application for a type of uncertainty problem. The paragon outcome is achieved in the scope of group work to compute the success value of the group. We hope that this research for the selection of the best possible parameter of a universal set in any soft set will be worthwhile to researchers working on problems having uncertainty in many features for further studies in this field.

***K*eywords** soft set, best possible parameter, distance function, algorithm, decision making

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ON SOME GAUSSIAN ORESME NUMBERS

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ABSTRACT

Sequences with recurrence relations are used in many branches of science such as mathematics, physics and engineering.The most well-known and studied sequence among these is Horadam sequence. This sequence is a generalization of many sequences and has an important place in the literature. New sequences can also be obtained from this sequence by changing the initial conditions. The most common sequences obtained by this method in the literature are Fibonacci sequence, Lucas sequence and Jacobstal sequence. The Fibonacci sequence, which has an important place among these sequences, has been studied by many authors. Horadam, who also worked on Fibonacci sequences, defined these sequences in complex space and discussed Gaussian Fibonacci sequences. A new sequence, which was defined by Nicole Oresme in the $14th$ century and called the Oresme sequence, is a special case of the Horadam sequence whose initial conditions are rational numbers. Subsequently, this was reviewed by many authors. In this study, we defined the Gaussian Oresme sequence using the Oresme sequence. We gave the Binet formula for Gaussian Oresme numbers by using the Binet formula of Oresme numbers. From here, we have obtained the recurrence relation and the generating function of this number sequence. We have given important identities of this sequence such as Cassini, Catalan, d'Ocagne's, Honsberger's and Vajda identities. Moreover, we defined the matrix of the Gaussian Oresme sequence and calculated the $n-th$ power. We have given the finite sum formula for Gaussian Oresme numbers. We have shown the relationship of this sequence with the Gaussian Fibonacci Numbers.

***K*eywords** Oresme Numbers · Gaussian Numbers · Reccurence Relations

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PEDAGOGICAL EXPERIENCE TO PROMOTE ACADEMIC SUCCESS: A CASE STUDY

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ABSTRACT

Failure in the curricular units of Differential and Integral Calculus (CU-DIC) and Introduction to Programming (commonly referred to as CS1) are realities that teachers face in undergraduate courses in Computer Engineering. Having a solid foundation in mathematics is crucial for programming, and there are studies that demonstrate a correlation between programming proficiency and experience in mathematics. The CU-DIC plays a pivotal role in providing the theoretical foundation required for future professionals in fields like engineering and other exact sciences. As a result, these units are included in the majority of degree programs offered at various higher education institutions. The high failure and dropout rates in DIC have prompted the need to critically evaluate the methodologies and teaching styles, the learning environments created, and the assessment practices. It is essential to establish educational practices that empower students to become co-responsible for their learning process, fostering their academic success and meaningful comprehension. Successful students should actively engage in strategies to motivate themselves, shape their behavior, and facilitate learning. Teachers can play a pivotal role in enhancing outcomes, utilizing teaching strategies that address the weaknesses of the class or individual students, as well as implementing learning strategies that leverage students' strengths. At the Coimbra Institute of Engineering (ISEC), teachers have applied several strategies that allow reversing the results found in the CU-DIC: CU operating in alternative semesters, support centers for mathematics, formative assessment, or use mathematical software in classes, among others. However, the impact of applying each of these strategies may be related to the learning styles of each student and the attitude that each one has towards the difficulties encountered in integrating into the CU-DIC, namely, in basic and elementary concepts in mathematics. In view of all these facts, in the academic year 2022/2023, a pedagogical experiment was carried out that tried to link programming skills with CDI learning. This communication intends to describe this experience, as a case study,

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relating it to the results obtained in CU-DIC of the Informatics Engineering degree at ISEC.

***K*eywords** curricular units · Differential and Integral Calculus · dropout rate

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ON A CLASS OF FOURTH ORDER RECURSIVE SEQUENCES

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ABSTRACT

The concept of recursive sequences is one of the most attractive topics in Mathematics, especially in Number Theory. A recursive sequence is defined by a recurrence relation and its initial conditions. Recurrence relations enable us to find subsequent terms by using previous terms. Some of the most known recursive sequences that are defined by second order recurrence relations are Fibonacci, Lucas, Pell, and Pell-Lucas sequences. Apart from these sequences, the sequence of balancing numbers, which is one of the second order recursive sequences, has been studied by many authors recently. The terms of the sequence of balancing numbers are the solution of the following equations

$$1 + 2 + \cdots + (n-1) = (n+1) + (n+2) + \cdots + (n+r) \tag{8}$$

By adding a fixed positive integer to the left side of this equation one can obtain the following equation that was introduced in [2]

$$1 + 2 + \cdots + (n-1) + D = (n+1) + (n+2) + \cdots + (n+r) \tag{9}$$

Solutions of these kinds of Diophantine equations which are the terms of certain fourth order recursive sequences are called subbalancing numbers.

In this study, we deal with subbalancing sequences that are defined by fourth order recurrence relations. Firstly, basic properties of these fourth order recursive sequences are presented. Then the relations between these fourth order recursive sequences and second order recursive sequences are investigated.

***K*eywords** recursive sequences · Diophantine equations · subbalancing numbers

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EFFECTS OF FEAR AND REFUGE USE BY PREY IN A PREDATION MODEL

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ABSTRACT

When there is a species interaction in an ecological environment, the fear of predation risk may provide the prey species privileges by limiting their exposure to possible predators. this action could result in the use of refuges by the prey population.

In earlier papers, different predation models have been modified considering prey refuge use and assuming that the population acts with fear. But, a partial analysis of the dynamics of systems has been made.

One of the purposes of this work is to point out that some of these new systems with such modifications can have the same topological portrait in the phase plane as the original one. In this way, there are no dynamic differences between the behaviors of the solutions of the models.

To attain the results, we first provide the existence and stability of the equilibria of the model. Next, we study the impact of the fear effect on the model analytically and numerically. Also, we compare the system with and without fear effect. Some simulations are shown to reinforce the theoretical results.

***K*eywords** Predator-prey · fear effect · refuge · stability

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THE EFFECT OF COVARIANCE STRUCTURES ON PARAMETER ESTIMATION UNDER MULTIVARIATE LAPLACE DISTRIBUTION IN MANOVA

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ABSTRACT

Repeated measures are multiple responses from the same experimental unit over time or sequentially under different trial conditions. Designs for these kind of data are frequently used in many disciplines such as medicine, psychology, and education for two factors, indicating between-subjects and within-subjects. In particular, the dependency structure, which models the relationship between different measures taken from the same experimental unit, emerges as an issue that should be considered in the analysis of data with repeated measures. Univariate and multivariate analysis approaches are used in the analysis of repeated measures data. While it is necessary to provide the assumption of sphericity in univariate approaches, this issue is more flexible in multivariate approaches. In multivariate approaches such as MANOVA and Mixed Model methods, different covariance structures can be used to model the dependency structure resulting from repeated measurements. In this study, it is aimed to examine the effect of different covariance structures with a simulation study for MANOVA parameter estimations. The simulation study is performed to compare the results of parameter estimations for different covariance structures under Multivariate Laplace distribution using the EM algorithm. Results are compared with respect to Euclidean distances.

***K*eywords** Covariance structure · Repeated MANOVA · Multivariate Laplace distribution · EM algorithm

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UNPREDICTABLE SOLUTIONS OF QUASILINEAR SYSTEMS WITH DISCONTINUOUS RIGHTHAND SIDES

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ABSTRACT

It is demonstrated that there exists a unique unpredictable solution of a quasilinear system whose righthand side is discontinuous such that it is constructed by means of an unpredictable sequence. The Lipschitz constant of the nonlinear term is required to be sufficiently small to achieve the main result. The stability of the unpredictable solution is investigated.

***K*eywords** Unpredictable solution · Discontinuous righthand side · Quasilinear systems · Stability

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UNPREDICTABILITY IN RETARDED SHUNTING INHIBITORY CELLULAR NEURAL NETWORKS

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ABSTRACT

In this study we investigate unpredictable oscillations occurring in a class of retarded shunting inhibitory cellular neural networks. Rectangular input currents are utilized in the model. The exponential stability of such oscillations is demonstrated, and the extension of unpredictability among coupled networks is discussed. Examples with computer simulations are presented.

***K*eywords** Shunting inhibitory cellular neural networks · Rectangular input currents · Unpredictable oscillations

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FORMS OF ASSESSMENT IN VIEW OF THE DEVELOPMENT OF MATHEMATICAL COMPETENCIES

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ABSTRACT

During the past twenty years, the need to define the outcomes of education as Competences has been recognized in many countries. The European Parliament and the Council of the European Union acknowledged the mathematical competence in 2006 to be a key competence for everyday life [4] and its partial competencies were incorporated into national educational programs. The engineering tertiary educational bodies grouped around The European Society for Engineering Education (SEFI) Mathematics Special Interest Group (MSIG) adopted the concept of eight overlaying mathematical competencies introduced by Danish KOM project [2] in 2003 which were linked to the carefully specified core content, and formulated as the core content-related competencies defining learning outcomes in the third edition of "A Framework for Mathematics Curricula in Engineering Education" [1]. The importance of developing "mathematical competence" for forming of mathematical literacy is nowadays recognized across all continents, even though differing a little in partial competencies classification. Albeit research papers and case studies on methods and processes of developing particular competencies are relatively abundant, the assessment possibilities with respect to particular competencies are not so frequent subject of research.

In paper, we will identify appropriate forms of summative assessment, applicable in the higher engineering education following up on the results presented in the proceedings of the conference ICMASE 2020 on New Rules for Assessing Mathematical Competencies [3]. In addition, the special attention will be paid to mutual communication in a class, a phenomenon that was negatively affected to a large extent by the pandemic restrictions. We will introduce a discussion as a part of summative assessment, not only providing the space for evaluating the competencies, but more, as an aspect that essentially contributes to their cultivation. Furthermore, the strategy of deploying assessment forms will be discussed, taking into account the building of comprehensive Mathematical Competency in gradual continuous and permanent manner.

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***K*eywords** Mathematical Competencies · Tertiary engineering education · Assessment forms · Discussion

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ON DETERMINANTS AND NORMS OF CIRCULANT MATRICES WITH FIBONACCI QUATERNIONS

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ABSTRACT

At this paper, we consider $\mathcal{T}_n := \text{circ}\,(Q_1, Q_2, \ldots, Q_n)$ circulant matrices whose entries are the well-known Fibonacci quaternions $Q_1, Q_2, \ldots, Q_n$. Then, we compute determinants of $\mathcal{T}_n$ by exploiting the set of orthogonal polynomial, Chebyshev polynomials of the second kind. Moreover, we obtain some kind of norms of these matrices.

***K*eywords** Circulant matrix · Fibonacci quaternion · Determinant · Norm

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G_2- FRAME FORMULAE FOR SPATIAL SPLIT OCTONIONIC CURVES

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ABSTRACT

In this presentation, we first define the vector product in Minkowski space $\mathbb{R}_4^7$ which is identified with the space of spatial split octonions. Then, we obtain the G_2- frame formulae for a seven dimensional Minkowski curve by using the spatial split octonions and the vector product.

***K*eywords** Vector Product · Minkowski space · Spatial Split Octonions · Curvatures

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ICMASE 2023

NEW G-CLOSED SETS WITH RELATED TO AN IDEAL

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ABSTRACT

In the mathematical field of set theory, an ideal or set ideal is a partially ordered collection of sets that are considered to be "small" or "negligible". On the other hand the notion of generalized closed (briefly, g-closed) sets was defined by Levine. Then, several authors studied on this subject. In this paper, we introduce new g-closed sets and investigate of it's basic properties using ideal.

***K*eywords** $g-$closed sets $\cdot$ $I-g-$closed sets $\cdot$ $*g-$closed sets, supra topology.

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ADAL CLASS OF SURVIVAL REGRESSION MODELS: AN APPLICATION TO RIGHT-CENSORED LIFETIME DATA SET

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ABSTRACT

One of the most crucial areas of statistics in biology and medicine is survival analysis. Additionally, full probabilistic modelling is essential in many real-world applications in various fields, including medical, engineering, economics, and social sciences, and the difficulties in survival models have changed significantly over the past few decades. The objective of this article is to introduce the "Adal" class, a new, general, tractable class for the widely used survival regression models. The Cox proportional hazard (PH) model was extended by the proposed class. Additionally, it specifies several new sub-models and includes numerous well-known models as sub-models, such as the proportional odds (PO), accelerated hazard (AH), accelerated odds (AO), and accelerated failure time (AFT) models. For the proposed class, we derive a number of probabilistic functions, and we establish a mathematical link with the survival regression models. For parameter estimation and inference, we used maximum likelihood technique. Finally, an actual dataset involving cancer patients with leukemia is thoroughly addressed to assess the versatility and utility of the proposed class.

***K*eywords** Survival analysis; proportional hazard model; accelerated odds model; Adal class; accelerated failure time model; maximum likelihood estimation; censored data; accelerated hazard model.

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CONNECTION OF BALANCING NUMBERS WITH SOLUTION OF A SYSTEM OF TWO HIGHER-ORDER DIFFERENCE EQUATIONS

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ABSTRACT

We provide some theoretical justifications pertaining to the representation for the solution of the system of the higher-order rational difference equations

$$x_{n+1} = \frac{1}{6 - y_{n-k}}, \quad y_{n+1} = \frac{1}{6 - x_{n-k}}, \qquad n, k \in \mathbb{N}_0.$$

where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, and the initial conditions x_{-k}, $x_{-k+1}, \ldots, x_0$, y_{-k}, $y_{-k+1}, \ldots$, y_0 are non zero real numbers such that their solution is related to Balancing numbers. We also study the stability character and asymptotic behavior of this system.

Keywords General solution · Balancing numbers · stability · System of difference equations.

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DELAMINATION RESISTANCE OF LAMINATED GLASS PLATES HAVING ETHYL VINYL ACETATE, POLYVINYL BUTYRAL AND SENTRYGLAS PLUS INTERLAYERS

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ABSTRACT

Laminated glass is manufactured by laminating two or more layers of glass together with a flexible interlayer at specified pressure and temperatures using an autoclave. The characteristic properties of laminated glass can be affected considerably by the used interlayers.Polyvinyl butyral, ethyl vinyl acetate and sentryglass are three main interlayers used by glass manufacturers. In order to select the appropiate interlayer parameters such as blast performance, availibity, cost, durability, optical property and manufacturing equipment should be considered. Usage of laminated glass is expanding due to its safety and security property as well as comfort and design. As a consequence of possess superiority of laminated glass for structural applications in various industries, they strengthen their position between building materials. Nevertheless delamination which may be result of manufacturing process and service is regarded the primary concern and most undesirable failure mode for the analysis of laminated glass.For this reason it is hot research area of composite industry. This paper presents the results of a study on the delamination strength of laminated glass with the mentioned three types of interlayers. A mathematical model is presented to analyze delamination behavior of laminated glass by using minimum potential energy theorem and applying variational method. Nonlinear equations which represent the laminated and delaminated regions of glass unit are obtained and then solved by applying finite difference method. Five nonlinear equations are written in matrix form. Solution of matrix system is obtained by using special matrix solvers and succesive over relaxation method is used to overcome the convergency problems.Deflection and stress values wwhich represent mechanical behavior of unit are presented in figures. In order to carry out validation assumptions of model the finite element model was constituted. ABAQUS is used to perform the finite element analysis. The finite element results are in a good match with results of developed mathematical model.

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***K*eywords** Laminated Glass · Delamination · Interlayer

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A SYMMETRIC EXPLORATION OF DUAL BALANCING AND COBALANCING NUMBERS AND QUATERNIONS

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ABSTRACT

This paper introduces Dual Balancing and Dual Cobalancing numbers, along with their corresponding quaternions, to unveil the hidden symmetry of quaternions. We present Binet's formulas, generating functions, and numerous intriguing properties that contribute to a comprehensive understanding of these concepts. Additionally, we provide matrix representations that offer a fresh perspective on Dual Balancing and Dual Cobalancing quaternions. The findings of this study extend beyond theoretical applications and may have practical implications in various fields

***K*eywords** Dual Balancing numbers · Dual Cobalancing numbers · Dual Balancing quaternions · Dual Cobalancing quaternions · Recurrence relations · Binet's formulas · Matrix representations

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FERMATEAN FUZZY TYPE A THREE-WAY CORRELATION COEFFICIENT

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ABSTRACT

A trustworthy information is metric for assessing the association between Fermatean fuzzy sets (FFSs) is the correlation coefficient (CC). There are a few methods for figuring out the CC of FFSs. These previous methodologies, which are specified between [0, 1], simply evaluate the strength of the link between FFSs. This study, using the notions of variance and covariance, respectively, provides a three-way method for computing the correlation coefficient (CC) between FFSs. The strength of the association between the considered FFSs is demonstrated by this novel approach, which is defined inside the interval [1, 1] similar to classical statistics, and it identifies whether the FFSs are positively or negatively associated. The potential of inaccuracy owing to information leakage is reasonably mitigated by the suggested technique's inclusion of the three traditional FFSs parameters. To demonstrate the new technique's usefulness as an accurate information measure, several theoretical results will be used to validate it. The advantages of the new methods over comparable methods will be demonstrated using a few numerical examples. With the help of the novel approach, several decision-making issues involving pattern recognition and diagnostic medicine will be handled. Multi-attribute decision-making issues can be resolved using the three-way technique for determining the correlation coefficient between FFSs.

***K*eywords** Fernatean fuzzy correlation coefficient · Medical decision-making · pattern recognition ·

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CHARACTERIZATION OF THE FINITE GROUPS

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ABSTRACT

In this article, I shall present the definitions of groups, subgroups, the finite groups, The Lagrange' s Theorem which tell us that the order of the subgroup H is the divisor of the order of the group G. This Theorem give us the possibility to find examples of subgroups of the finite groups. For example, the groups of order 4 has two subgroups: the cyclic group: $C_4 = \{1, z, z_2, z_3\}$ and Klein group.

Let G be a group of order 4. We assume that there is an element $x \in G$ with $x^2 \neq 1$. We can't have that $x^3 = 1$ because, in thie case, $\{1, x, x^2\}$ would be a subgroup of order 3 of G, contrary to Lagrange's Theorem.

Thus, $x^3 \neq 1$ and obvious $x^3 \neq x$, $x^3 \neq x^2$. This shows that $G = \{1, x, x^2, x^3\} =< x >$, so G is a cyclic group.

We now assume that for any element $x \in G$ with $x^2 = 1$. Let $x, y \in G$, $x \neq 1$, $y \neq 1$ and $x \neq y$.

- If $xy = x \implies y = 1$, but $y \neq 1$.
- If $xy = y \implies x = 1$, but $x \neq 1$.
- If $xy = 1$, we have $x = x1 = xy^2 = xyy = 1y = y$, but $x \neq y$.

So there is also the element xy. Now $G = \{1, x, y, xy\}$.

From $(xy)^2 = 1$, we have $(xy)(xy) = 1$.

Also, $(xy)(xy)^{-1} = 1$. Therefore, $xy = (xy)^{-1}$.

Then,

$$yx = (xy)^{-1} = xy.$$

We have the relations $x^2 = 1$, $y^2 = 1$ and $xy = yx$, which shows us that the multiplication table of G coincides with the multiplication table of the dihedral group D_2. It is defined as the symmetry group of a rectangle that is not square. Noting σ and τ symmetries in relation to the two symmetry axes of the rectangle, $\sigma \circ \tau = \tau \circ \sigma$ is the symmetry with respect to the center of the rectangle.

Then $D_2 = \{1, \sigma, \tau, \sigma \circ \tau\}$ with $\sigma^2 = 1$, $\tau^2 = 1$, $\sigma \circ \tau = \tau \circ \sigma$. This is usually called the Klein group (it is the smallest group that is not a cyclic group).

$G \simeq C_4$, because they are cyclic groups with the same order, where $C_4 = \{z \in \mathbb{C} | z^4 = 1\}$. In other words, $C_4 = \{1, z, z^2, z^3\}$.

Using the same reasoning, the groups of order 6 has also two subgroups: $S_3 = \{1, \sigma, \sigma^2, \tau, \sigma \circ \tau, \sigma^2 \circ \tau\}$, it can be realized as the group of rotations and reflections of an equilateral triangle, and $C_6 = \{z \in \mathbb{C} | z^6 = 1\}$.
Also, the groups of order 8 can be:

$$G \simeq \mathbb{Z}_8; \quad G \simeq \mathbb{Z}_4 \times \mathbb{Z}_2 \quad G \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2;$$

$$G \simeq D_8 \quad G \simeq \mathbf{Q},$$

where the dihedral group D_8 is the symmetry group of the square and the quaternion group $\mathbf{Q}$ has the same order as the dihedral group D_8, but a different structure, as shown by their Cayley table. In this two cases, the group G is not abelian.

***K*eywords** Groups · Subgroups · Finite groups · Lagrange's Theorem

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