VII International Conference on Mathematics and its Applications in Science and Engineering
(ICMASE 2026)

QUDIT QUANTUM COIN FLIPPING AND BINET FORMULAS FOR GENERALIZED FIBONACCI NUMBERS

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ABSTRACT

The quantum coin flipping problem for the generic qudit coin, with arbitrary number d of maximally random states is studied. For a given d we construct the set of orthonormal quantum coins, generated by the Hadamard gate and determined by the roots of unity. The states are the eigenstates of the number operator $N = \sum_k^{d-1} k|k\rangle\langle k|$ and of the Sylvester clock operator $\Sigma_3 = q^{2N}$, $q^{2N}|k\rangle = q^{2n}|k\rangle$, acting on the coin states as the shift operator. The clock and shift operators are the q-commutative operators, generating the quantum group symmetry for the qudit coin states. For flipping quantum coin the probability of duplicated states in a fixed number of trials is represented by generalized Fibonacci numbers. For these numbers the Binet formulas are derived. It is shown that they represented by two parametric pq-quantum calculus numbers and in particular cases, they include several sequences of numbers such as, the Mersenne numbers, the Fibonacci-Lucas numbers, and the Pell numbers. In particular, the probability for duplicated states of $d = 5$ ququit quantum coin,

$$P_n = \frac{1}{4}\left(\frac{2}{5}\right)^n \mathcal{P}_{n-1},$$

is determined by numbers

$$\mathcal{E}_n = 2^{n-1}\frac{(1+\sqrt{2})^n - (1-\sqrt{2})^n}{2\sqrt{2}} = 2^{n-1}\mathcal{P}_n,$$

where $\mathcal{P}_n = 0, 1, 2, 5, 12, 29, 70, ...$ are the Pell numbers. In case of quhexit coin with six basis states we have probability of duplicated states, determined by the Binet formula with the golden ratio $\varphi = (1+\sqrt{2})/2$ and $\varphi' = -\varphi^{-1}$,

$$H_n = 5^{\frac{n-1}{2}}\frac{\varphi^{2n} - (-1)^n \varphi'^{2n}}{3}.$$

For the even and odd index n, this formula can be rewritten in terms of Fibonacci numbers F_n and Lucas numbers L_n,

$$\begin{aligned} H_{2k} &= \frac{5^k}{3} F_{4k}, \\ H_{2k+1} &= \frac{5^k}{3} L_{4k+2}. \end{aligned}$$

Finally, we show that every n-qubit state $|\psi_k\rangle$ represents a qudit quantum coin with number of states $d = 2^n$ and the number of such coins is equal 2^n. In this case, the Binet formula is

$$D_N = \left(\frac{\sqrt{M_n}}{2}\right)^N \frac{(\sqrt{M_n} + \sqrt{M_n+4})^N - (\sqrt{M_n} - \sqrt{M_n+4})^N}{\sqrt{M_n(M_n+4)}}$$

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where

$$M_n = [n]_2 = \frac{2^n - 1}{2 - 1} = 2^n - 1,$$

are the Mersenne numbers.

Keywords Quantum coin · Binet formula · Fibonacci numbers · qudit states

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