

HIGHER ORDER COMPACT PARTITIONED AVERAGE VECTOR FIELD METHOD FOR KLEIN-GORDON-SCHRÖDINGER EQUATION

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ABSTRACT

The nonlinear Schrödinger equation coupled with the nonlinear Klein–Gordon equation is known as a classical coupled nonlinear Klein–Gordon–Schrödinger (KGS) system to describe a system of conserved scalar nucleons interacting with neutral scalar mesons where the dynamics of these fields are coupled through the Yukawa interaction [1]. The system plays an important role in quantum physics. The KGS system is a nonlinear system of partial differential equation which does not have exact solution. Therefore numerical studies are essential to solve the system. The KGS system can be cast into a Hamiltonian form which possess the energy conservative law as well as mass conservation law [2]. Standard numerical discretization such as 4th-order Runge-Kutta method does not preserve these conservative properties of the system and destroys the qualitative behavior of the system.

In this study, we design a linearly implicit higher order compact (HOC) conservative numerical scheme which preserves the original conservative properties to solve the KGS equation. The proposed scheme is based on using the higher order compact (HOC) finite difference method [3] and the partitioned average vector field (PAVF) schemes [4]. At first, we discretize the Hamiltonian system in space by a HOC method which has higher convergent rate than general finite difference methods. Then the semi-discretized system is approximated in time by the PAVF fields method which preserves the Hamiltonian structure as well as the mass structure of the original system. The conserved quantities of the scheme such as energy and mass conservation law are discussed. We obtain that the new scheme preserves the energy and mass of the system very well in long term integration.

Keywords Klein–Gordon–Schrödinger system \cdot compact scheme \cdot partitioned average vector field method \cdot conservative scheme

References

- [1] Fukuda I., Tsutsumi M., On coupled Klein–Gordon–Schrödinger equations II, J. Math. Anal. Appl., 66, 358–378, 1973.
- [2] Kong L., Chen M., Yin X., A novel kind of efficient symplectic scheme for Klein–Gordon–Schrödinger equation, A pplied Numerical Mathematics , 135, 481–496, 2019.
- [3] Pan X., Zhang L., High-order linear compact conservative method for the nonlinear Schrödinger equation coupled with the nonlinear Klein–Gordon equation, Nonlinear Analysis, 92, 108–118, 2013.
- [4] Cai W. Li H., Wang Y., Partitioned averaged vector field methods, Journal of Computational Physics, 370, 25–42, 2018.

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