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DIOPHANTINE TRIPLES IN LINEAR RECURRENCES

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ABSTRACT

A Diophantine $m-$ tuple is a set of m distinct positive integers $\{a_1, a_2, \ldots, a_n\}$ such that $a_i a_j + 1$ is a square for all $1 \leq i < j \leq n$. If $m = 3$, it is called Diophantine triples. For example $\{1, 3, 8\}$ is a Diophantine triples which are consecutive Fibonacci numbers. The Fibonacci numbers satisfy the recurrence relation

$$F_n = F_{n-1} + F_{n-2}$$

for ≥ 2 with initials $F_0 = 0$ and $F_1 = 1$. More generally, the set $\{F_{2n}, F_{2n+2}, F_{2n+4}\}$ is a Diophantine triples. The companion sequence of Fibonacci is known Lucas sequence $\{L_n\}$ safisfies the same recurrence with initial conditions are $L_0 = 2$ and $L_1 = 1$. The equation system

$$\begin{aligned} ab + 1 &= E_x \\ ac + 1 &= E_y \\ bc + 1 &= E_z \end{aligned}$$

was solved by Luca ans Szalay where E_n is n^{th} Fibonacci and Lucas number. In this talk, we give the details about other linear recurrences and Diophantine triples.

***K*eywords** Diophantine triples, linear recurrence, Fibonacci numbers with generalizations.

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