PERTURBED MODULAR HAUSDORFF INTERPOLATIVE IFS AND FRACTAL ATTRACTOR STABILITY

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Abstract

This study presents a modular Hausdorff interpolative approach to fractal attractors. In contrast to the classical Hutchinson construction, the ordinary Hausdorff distance is replaced by a scale-dependent modular Hausdorff distance. For a finite family of modular contraction mappings $T_1, T_2, \dots, T_N$, we study the modular Hutchinson operator

$$\mathcal{F}(A) = \bigcup_{i=1}^{N} T_i(A).$$

Under appropriate completeness and contractive assumptions, we prove the existence and uniqueness of a compact attractor $\mathcal{A}^*$ that satisfies $\mathcal{F}(\mathcal{A}^*) = \mathcal{A}^*$. Moreover, the Picard iteration generated by $\mathcal{F}$ converges to this attractor for every admissible initial compact set.

The second part of the study deals with perturbation stability. We explain that small modular Hausdorff perturbations of the Hutchinson operator lead to controlled deviations between the corresponding attractors. This gives a scale-dependent stability estimate for fractal generation and provides a modular extension of the classical fractal attractor theory. A Sierpiński-type modular construction is included to illustrate the applicability of the result.

***K*eywords** modular metric space · Hausdorff interpolative metric · Hutchinson operator · iterated function system · fractal attractor · stability

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