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DISCOVERING POINT GROUPS BY USING MODULAR ORIGAMI

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ABSTRACT

It is well-known that there exist numerous connections between origami and mathematics, from basic geometrical shapes, over geometric constructions using paper-folding, combinatorics and graph theory, up to abstract algebra and number theory [4, 6, 7]. The idea to use origami in teaching and popularizing mathematics and chemistry is natural, in particular at the level of primary education. Still, research in this area is scarce and so far has been limited to teaching basic geometry [2, 5, 10], effects on mathematical thinking in general [11], and teaching visualisation of chemical structures [3]. Some of these studies also include some symmetry considerations, but mostly limited to mirror (reflection) symmetry.

On the other hand, the theory of symmetry groups is fundamental for crystallography (and stereochemistry). Symmetry group theory has in fact historically developed from crystallographic considerations of symmetries of crystals and crystal structures [1]. Macroscopic crystals (as well as VSEPR models of molecule geometry) are modelled by various polyhedra with huge importance of their point symmetry groups, and crystal structures can be modelled by periodic tesselations and classified according to symmetry into space groups. The usual way to teach the necessary symmetry theory is using cardboard models, so the idea that origami could be used to teach symmetry is quite obvious.

Modular origami is an origami technique in which the final model is assembled from parts (modules) which are usually much easier to fold than the usual origami models. It allows a variety of examples of polyhedra with different types of point symmetry to be folded quickly, cheaply and easily [9], and has several advantages with respect to the usual cardboard and other solid models. In this presentation, we aim to introduce a method to teach fundamental point group theory concepts (recognition of various types of symmetry, the notion of point groups and their subgroups) using modular origami. This shall be done in an interdisciplinary fashion, combining mathematical abstract thinking and geometrical reasoning with its applications in crystallography and stereochemistry. More specifically, the ideas of the approach shall be demonstrated by making the well-known modular origami model known as Jackson cube [8], first in one color to investigate the full point group of the cube (O_{h}), and then by exchanging modules make it multicoloured to discover its subgroups.

***K*eywords** mathematics education · symmetry · origami · mathematical crystallography · mathematical chemistry · hands-on activities in teaching mathematics · group theory · point groups

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