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# HIGHER-ORDER RECURRENCES IN SCIENCE AND ENGINEERING EDUCATION: FIBONACCI AND TRIBONACCI APPROACHES TO ALGORITHMIC THINKING

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Sandra Ricardo<sup>1,\*</sup>, Paula Catarino<sup>1</sup>, Eudes Antonio Costa<sup>2</sup>

<sup>1</sup>University of Trás-os-Montes e Alto Douro, Vila Real, Portugal

<sup>2</sup>Federal University of Tocantins, Arraias, Tocantins, Brazil

## ABSTRACT

Linear recurrence sequences constitute an important class of mathematical objects, with applications in mathematics, computer science, and engineering. They provide canonical models for discrete dynamical systems and play a central role in the study of recursive processes, difference equations, and discrete structures. This work revisits the Fibonacci sequence, introduced by Leonardo of Pisa in the Liber Abaci (1202), and its higher-order generalisation, the Tribonacci sequence, formalised in the twentieth century, as a unifying framework that articulates mathematical foundations, modern applications and educational practice. We review the recurrence relations, characteristic equations and Binet-type closed forms of both sequences, and discuss their matrix representations, in which the golden ratio and the Tribonacci constant appear as the dominant eigenvalues of the corresponding  $2 \times 2$  and  $3 \times 3$  companion matrices, respectively. Building on these algebraic foundations, we propose a structured didactic pathway - from the classical rabbit problem to the generalised Tribonacci model and its algorithmic implementation - designed to foster algorithmic and computational thinking in mathematics, science and engineering education. The transition from second- to third-order recurrence is presented not as a mere algebraic extension, but as a conceptual leap supporting modelling competencies, abstraction and recursive reasoning, with direct relevance for the training of future scientists and engineers.

**Keywords** Fibonacci sequence · Tribonacci sequence · higher-order linear recurrences · Matrix representation · Algorithmic thinking · Mathematics education for science and engineering

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\*Corresponding Author's E-mail: sricardo@utad.pt