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ON T-SUPPLEMENTED MODULES

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ABSTRACT

In this work, all rings have identities and all modules over a ring R are unitary left $R-$modules. Let M be an $R-$module and $N \leq M$. If $L = M$ for every submodule L of M such that $M = N + L$, then N is called a small (or superfluous) submodule of M and denoted by $N \ll M$. A submodule N of an $R-$module M is called an essential submodule and denoted by $N \trianglelefteq M$ in case $K \cap N \neq 0$ for every submodule $K \neq 0$, or equvalently, $N \cap L = 0$ for $L \leq M$ implies that $L = 0$. Let M be an $R-$module and $U, V \leq M$. If $M = U + V$ and V is minimal with respect to this property, or equivalently, $M = U + V$ and $U \cap V \ll V$, then V is called a supplement of U in M. M is said to be supplemented if every submodule of M has a supplement in M. If every essential submodule of M has a supplement in M, then M is called an essential supplemented (briefly, e-supplemented) module. Let M be an $R-$module. The radical of M is defined by the intersection of all maximal submodules of M and denoted by $RadM$. If M have no maximal submodules, then the radical of M is defined by $RadM = M$. Let M be an $R-$module. If every submodule of M which contains $RadM$ has a supplement in M, then M is called a strongly radical supplemented module. Let M be an $R-$module and $T \leq M$. M is called a T-supplemented module if every submodule of M which contains T has a supplement in M. In this work, some properties of T-supplemented modules are investigated. Let M be an $R-$module and $T \leq M$. If M is supplemented, then clearly we can see that M is T-supplemented. Because of this T-supplemented modules are more general than supplemented modules. Let M be an $R-$module and $T \leq M$. If M is T-supplemented and $T = RadM$, then M is strongly radical supplemented.

***K*eywords** Small Submodule · Radical · Supplemented Module · r-Supplemented Module

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