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CYLINDRICAL REFLECTED ORNSTEIN–UHLENBECK MODEL FOR HIGH-SPEED MOBILE NETWORKS

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ABSTRACT

High-speed trains and motorway platoons form quasi-one-dimensional corridors in which user equipment experiences both rapidly varying path loss and slow correlated shadowing. To capture these phenomena in a unified framework, we introduce a two-dimensional reflected Ornstein–Uhlenbeck diffusion

$$X_t = (Q_t, S_t) \in [q_{\min}, q_{\max}] \times S^1,$$

where S_t is the periodic position of the mobile receiver within a cell and Q_t evolves according to

$$dQ_t = -\alpha\big(Q_t - q(S_t, v_0)\big)\,dt + \sigma\,dW_t + dL_t, \quad dS_t = v_0\,dt \pmod 1,$$

with reflecting boundaries at $q_{\min}$ and $q_{\max}$. The deterministic profile

$$q(s, v_0) = q_0 - a\,d_\varepsilon(s)^\gamma - b\,v_0 - c\ln(1 + v_0)$$

combines a smooth wrap-around path-loss term and a Doppler-induced penalty.

For each fixed speed $v_0 > 0$, this Markov process admits a unique smooth stationary density $\pi_{v_0}(q, s)$ on the cylinder. Up to normalization,

$$\pi_{v_0}(q, s) = \int_0^{1/v_0} \exp\Big\{\tfrac{2\alpha}{\sigma^2}\int_{q_{\min}}^{q}\big(q' - q(s + v_0\tau, v_0)\big)\,dq'\Big\}\,d\tau.$$

This explicit form allows computation of performance metrics without resorting to simulation.

When a single-server queue with Poisson arrivals of rate λ_{in} is coupled to the channel so that the service rate is a function $u(Q_t)$ of the instantaneous signal level, one finds that the joint process of queue length and channel state is positive recurrent if and only if

$$\lambda_{\text{in}} \;<\; \overline{\mu}(v_0) \;=\; \int_{S^1}\int_{q_{\min}}^{q_{\max}} u(q)\,\pi_{v_0}(q, s)\,dq\,ds.$$

This condition provides a clear buffer-dimensioning rule for high-speed scenarios.

By combining wrap-around geometry, reflecting signal bounds, Doppler-aware drift and closed-form stationary analysis, this model furnishes a compact and analytically tractable tool for performance evaluation of mobile aggregators in high-speed rail and highway networks.

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