VII International Conference on Mathematics and its Applications in Science and Engineering (ICMASE 2026)

ICMASE 2026

THE RIEMANN ZETA FUNCTION AND QUANTUM CALCULUS OF PRIMES

Oktay K. Pashaev*

Department of Mathematics, Izmir Institute of Technology, Izmir, 35430 Türkiye

ABSTRACT

In this paper we propose some applications of quantum q-calculus to several problems related with the Riemann zeta function. For removing the pole singularity for this function at $s = 1$, we first apply the Möbius transformation, moving it to infinity. Then, we formulate and solve the q-differential equation

$$xD_q f(x) = \left[\frac{1}{2}\right]_q f(x).$$

for nontrivial zeros of zeta function. The general solution of this equation takes the form

$$f(x) = \sqrt{x}\, G_q(x),$$

where $G_q(x)$ is the q-periodic modulation function, satisfying

$$D_q G_q(x) = 0$$

and represented in the general form as

$$G_q(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\beta \ln x}.$$

This suggests that the quantum Hamiltonian operator for solution of the Riemann problem could be based on the q-deformed version of the Berry-Keating Hamiltonian operator in the form

$$H = xD_q + D_q x.$$

Finally, we derive the Planck type formula for the gas of prime numbers in terms of q-numbers,

$$\langle E\rangle_N = \hbar\omega q \frac{d}{dq} \ln[N+1]_q$$

representing the truncated Riemann zeta function as

$$\zeta_N(s) = \frac{1 - p_1^{-s(N_1+1)}}{1 - p_1^{-s}} \frac{1 - p_2^{-s(N_2+1)}}{1 - p_2^{-s}} \ldots \frac{1 - p_l^{-s(N_l+1)}}{1 - p_l^{-s}}.$$

In the limit $N \to \infty$ it gives the Euler infinite product representation of zeta function

$$\zeta(s) = \prod_{primes} \frac{1}{1 - \frac{1}{p^s}},$$

as the Planck formula for gas of prime numbers.

Keywords Riemann zeta function · quantum calculus · Planck formula · prime numbers · Berry-Keating Hamiltonian

Corresponding Author's E-mail: oktaypashaev@iyte.edu.tr

References

[1] Berry M.V., Riemann's zeta function: A model for quantum chaos? In Quantum Chaos and Statistical Nuclear Physics. T. H. Seligman and H. Nishioka (eds), Lecture Notes in Physics, 263 Springer-Verlag, New York, 1986.

[2] Berry M.V. and Keating J.P., H=xp and the Riemann zeros. In Supersymmetry and Trace Formulae: Chaos and Disorder, edited by I.V. Lerner et al. Kluwer Academic/ Plenum: New York, 1999.

[3] Pashaev O.K., Quantum Calculus and Factorization of Fibonacci number. In Mathematical Methods for Engineering Applications. S. Gocheva-Ilieva et al. (eds.), Springer Proceedings in Mathematics and Statistics, 554, 2026.

[4] Julia B., Statistica Theory of Numbers. In Number Theory and Physics. J.M. Luck et al. (eds.), Springer Proceedings in Physics, 47, 1990.