
A NEW OPERATOR CLASS ASSOCIATED WITH L-WEAKLY AND M-WEAKLY COMPACT OPERATORS

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ABSTRACT

The operator classes L -weakly and M -weakly compact were introduced by P. Meyer-Nieberg. It was established that these classes are subsets of weakly compact operators, yet they do not imply compactness, nor do compact operators necessarily fall into these categories. Specifically, a non-empty bounded subset A of a Banach lattice E is defined as L -weakly compact if $\|x_n\| \rightarrow 0$ as $n \rightarrow \infty$ for every disjoint sequence (x_n) contained within the solid hull of A . A bounded linear operator T from a Banach space X to E is said to be L -weakly compact if $T(B_X)$ is L -weakly compact in E , where B_X represents the closed unit ball in X . Conversely, a bounded linear operator from E to X is M -weakly compact if $\|Tx_n\| \rightarrow 0$ as $n \rightarrow \infty$ for every disjoint sequence (x_n) in B_E .

In contrast to the complexities of analyzing weakly compact operators within a general Banach lattice framework, the subclasses of L -weakly and M -weakly compact operators, which are specific types of weakly compact operators, exhibit a range of intriguing properties. Extensive literature addresses the interactions between L -weakly and M -weakly compact operators and their connections with weakly compact operators and other operator classes. Although this operator class does not exist, it has been established that the class of regular L -weakly and regular M -weakly compact operators is in the form of a Banach lattice and possesses many attractive ordered properties.

Recently, several new classes of operators related to L -weakly and M -weakly compact operators have been introduced, along with their associated properties. These include LW -compact operators, almost L - and M -weakly compact operators, null almost L -weakly compact operators, and un L -weakly and un M -weakly compact operators. These operator classes are generalized new classes of L -weakly and M -weakly compact operators. In this study, we have introduced a new class of operators termed "LM-compact operators," which is defined in relation to L -weakly and M -weakly compact operators. It was found that this new class of operators retains some of the ordered properties of the original class, while lacking others. Additionally, comparisons with compact and weakly compact operator classes, as well as some of its algebraic properties and duality properties, were also investigated.

Keywords Banach lattice · L -weakly compact · M -weakly compact

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