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NON-AUTONOMOUS EULER–LAGRANGE ODES VIA TEMPERED RIEMANN–LIOUVILLE FRACTIONAL-DERIVATIVE-TYPE VARIATIONS

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ABSTRACT

In this paper, we provide new contributions to variational analysis and optimal control theory by addressing a new class of optimal control models based on tempered Riemann– Liouville fractional-derivative-type variations. Specifically, we formulate an optimal control problem equipped with a C^1-class controlled Lagrange functional, a simple integral objective functional, and a set of ordinary differential equations (ODEs) as first-order constraints. Thereafter, we recall appropriate variational techniques and design the perturbed functions using tempered Riemann– Liouville fractional derivatives of order $\alpha \in (n-1, n], n \in \mathbb{N}$. Notably, we reconsider the integration-by-parts formula within a fractional setting. In this context, we develop a variational framework yielding the non-autonomous Euler-Lagrange ODEs. To the best of our knowledge, this is the first study to incorporate fractional-derivative-type variations. As a result, the inclusion of tempered fractional-derivative-type variations leads to a broader class of Euler-Lagrange equations, capable of capturing memory and nonlocal effects that are not accounted for by the classical variational framework.

***K*eywords** Tempered fractional derivatives · Non-autonomous Euler–Lagrange equations · Optimal control theory · Riemann–Liouville fractional calculus

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