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NORM-DEPENDENT OVERSHOOT BOUNDS FOR MULTIDIMENSIONAL RANDOM WALKS WITH APPLICATIONS TO PORTFOLIO RISK

Bogdan M. Kushnarenko[1,2,3*], Elmira Yu. Kalimulina[3]

[1]*Lomonosov Moscow State University, Moscow, Russia*
[2]*VEGA Institute, Moscow, Russia*
[3]*Institute for Information Transmission Problems, RAS, Moscow, Russia*

ABSTRACT

Capital and liquidity buffers must cover not only the first time a cumulative loss process breaches a limit t, but also the *overshoot* R_t beyond that limit. When losses arise from several independent factors, a vector random walk is appropriate; yet classical Lorden bounds are scalar. We obtain explicit, dimension-free bounds for $\mathbb{E}R_t$ under an arbitrary norm, giving stress-test margins for multi-asset portfolios.

Model. Let $Z_1, Z_2, \dots \in \mathbb{R}^d$ be i.i.d. vectors with non-negative components, finite second moment and mean $m = \mathbb{E}Z_1 \neq 0$. Set $S_n = \sum_{k=1}^{n} Z_k$ ($S_0 = 0$). For a threshold $t > 0$ define the first-exit index and overshoot

$$N(t) = \inf\{n \geq 1 : \|S_n\| > t\}, \qquad R_t = \|S_{N(t)}\| - t.$$

Main bound. Fix any norm $\|\cdot\|$ on $\mathbb{R}^d$ and let $C > 0$ satisfy $C\|x\|_1 \leq \|x\|$ for all x. Then, for every $t > 0$,

$$\mathbb{E}R_t \ \leq \ \left(\tfrac{1}{C} + \tfrac{1}{\|\mathbf{1}\|}\right) \frac{\mathbb{E}\|Z_1\|^2 + \mathbb{E}\|Z_1\|_1^2}{\mathbb{E}\|Z_1\|_1}, \qquad \mathbf{1} = (1, \dots, 1)^\top.$$

The right-hand side is independent of the level t and the dimension d.

Idea of proof. Project S_n onto m to obtain a one-dimensional martingale to which the scalar Lorden bound applies; the orthogonal component is then bounded using Cauchy–Schwarz and the constant C. Summing the two contributions yields the stated inequality.

Euclidean-norm corollary. For $\|\cdot\| = \|\cdot\|_2$ one can take $C = d^{-1/2}$ and $\|\mathbf{1}\| = \sqrt{d}$, giving

$$\mathbb{E}R_t \ \leq \ 2\big(\sqrt{d} + d^{-1/2}\big) \frac{\mathbb{E}\|Z_1\|_1^2}{\mathbb{E}\|Z_1\|_1}.$$

Risk interpretation. The formula provides a closed-form capital add-on that depends only on first- and second-moment estimates of single-period factor magnitudes. Because neither t nor d appears, the same buffer protects thresholds across all horizons and dimensions, greatly simplifying multi-asset risk management.

[*]*Corresponding Author's E-mail: eyk@iitp.ru*