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# A MATHEMATICAL MODEL FOR SUSTAINABLE MULTI-ITEM INVENTORY SYSTEMS WITH DEMAND DYNAMICS, DETERIORATION, AND CARBON EMISSIONS

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## ABSTRACT

Inventory management plays a critical role in modern supply chains, particularly for products that are subject to deterioration and whose demand changes over time. In addition, growing environmental concerns have increased the need for sustainable inventory policies that not only minimize operational costs but also reduce environmental impacts. This research focuses on developing mathematical models for sustainable multi-item inventory systems that integrate demand dynamics and carbon emissions into inventory decision-making. The proposed models consider inventory-dependent and time-dependent demand patterns, which are commonly observed in perishable products such as fruits, vegetables, dairy products and meat. Inventory-dependent demand reflects the influence of stock availability on customer purchasing behavior, while time-dependent demand captures changes in demand over the product lifecycle. Furthermore, the models incorporate carbon emissions generated from storage and transportation activities, enabling a comprehensive evaluation of both economic and environmental performance. We consider several replenishment policies, including individual ordering, joint replenishment, and hybrid approaches that combine elements of both. The objective is to determine the optimal replenishment interval and the inventory level at which inventory drops to zero that minimizes total inventory cost while accounting for environmental considerations. Analytical techniques are used to derive optimality conditions and investigate the structural properties of the models. We also develop an optimization algorithm to obtain optimal solutions. Sensitivity analysis is also conducted to assess the impact of changes in parameters on the model's optimal replenishment time, the time at which inventory drops to zero, and the total inventory cost. The expected outcomes of this research include new mathematical formulations for sustainable multi-item inventory systems, the structure of optimal inventory policies, and practical solution algorithms. By integrating demand dynamics, deterioration, and carbon emissions, this research advances green inventory systems that balance economic efficiency with environmental sustainability.

**Keywords** Multi-Item Inventory Systems · Deteriorating Items · Demand Dynamics · Carbon Emissions · Sustainable Inventory Optimization

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