

GREATEST COMMON DIVISORS OF SHIFTED FIBONACCI AND LUCAS SEQUENCES

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ABSTRACT

Shifted Fibonacci sequences, obtained by adding a fixed integer a to the Fibonacci sequence, were first introduced in the literature in 1973. Although the greatest common divisor (gcd) of any two Fibonacci numbers is always a Fibonacci number, consecutive Fibonacci and Lucas numbers are relatively prime. Consequently, determining the gcd of consecutive shifted Fibonacci or Lucas numbers constitutes a special case worthy of investigation. Moreover, a variety of approaches, including different proof techniques and computational software packages, have been employed in the study of greatest common divisors. It was shown that consecutive shifted Fibonacci numbers are not necessarily relatively prime and that the values of $\gcd(F_n + a, F_{n+1} + a)$ are unbounded when $a = \pm 1$. More recently, the gcd properties of shifted Fibonacci and Lucas numbers have been studied extensively, with particular emphasis on the cases $a = \pm 1, \pm 2$, and ± 3 . In this study, we investigate the greatest common divisors of consecutive shifted Fibonacci and Lucas numbers. Our results show that, unlike the case $a = \pm 1$, the greatest common divisors of consecutive shifted Fibonacci and Lucas numbers are bounded.

Keywords Fibonacci numbers, Lucas numbers, shifted Fibonacci sequence, shifted Lucas sequence, greatest common divisor

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