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## A BRIEF STUDY ON THE TRIDIMENSIONAL LUCAS-COBALANCING NUMBERS

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### ABSTRACT

Currently, several sequences of numbers have been investigated by many researchers. One of these sequences is the Lucas-cobalancing numbers  $\{c_n\}_{n \geq 1}$  which are obtained through recurrence relations

$$B_{n+2} = 6B_{n+1} - B_n,$$

with initial values  $B_0 = 0$  and  $B_1 = 1$ , and

$$c_{n+2} = 6c_{n+1} - c_n,$$

with initial values  $c_1 = 1$  and  $c_2 = 7$ , respectively. (See for more information the works [1, 2, 5, 6, 9, 10], among others).

Many other versions of sequences have also been studied, such as the bidimensional versions of balancing  $\{B_{(n,m)}\}_{n,m \in \mathbb{N}}$  and Lucas-cobalancing  $\{c_{(n,m)}\}_{n,m \in \mathbb{N}}$  which are given by the recurrence relations

$$\begin{cases} B_{(n+1,m)} &= 6B_{(n,m)} - B_{(n-1,m)}, \\ B_{(n,m+1)} &= 6B_{(n,m)} - B_{(n,m-1)}, \end{cases}$$

with initial conditions  $B_{(0,0)} = 0$ ,  $B_{(1,0)} = 1$ ,  $B_{(0,1)} = i$ ,  $B_{(1,1)} = 1 + i$  and  $i^2 = -1$ , and

$$\begin{cases} c_{(n+1,m)} &= 6c_{(n,m)} - c_{(n-1,m)}, \\ c_{(n,m+1)} &= 6c_{(n,m)} - c_{(n,m-1)}, \end{cases}$$

with initial conditions  $c_{(0,0)} = 1$ ,  $c_{(1,0)} = 7$ ,  $c_{(0,1)} = i$ ,  $c_{(1,1)} = 7 + i$  and  $i^2 = -1$ , respectively. (For more information, see, among others, [3, 4]).

In this work, we introduce the tridimensional version of Lucas-cobalancing numbers. We also study some properties and some sum identities.

**Keywords** Balancing numbers · Lucas-cobalancing numbers · Tridimensional version · Properties

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