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THE SPECTRAL STRUCTURE OF ROOTS OF $\mathcal{M}$–STURM-LIOUVILLE PROBLEMS

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ABSTRACT

In this study, classical Sturm–Liouville problems are investigated within the framework of fractional calculus by employing the $\mathcal{M}$-derivative. To establish a mathematically rigorous and physically meaningful basis for the analysis, the one-dimensional Schrödinger equation is formulated as an $\mathcal{M}$–Sturm–Liouville problem. The primary objective of the study is to examine the eigenvalue–eigenfunction relationships arising in $\mathcal{M}$–Sturm–Liouville problems, with particular emphasis on the distribution of the zeros of the corresponding eigenfunctions.

One of the main contributions of this work is the interpretation of the zeros of the eigenfunctions as a nested network structure. Furthermore, two fundamental results of classical Sturm–Liouville theory, namely the Sturm Oscillation Theorem and the Sturm Comparison Theorem, are extended to the $\mathcal{M}$-derivative framework. These generalizations provide a theoretical foundation for the investigation of the oscillatory behavior of solutions and for the comparison of eigenfunctions associated with different $\mathcal{M}$–Sturm–Liouville problems.

Keywords Fractional · $\mathcal{M}$-Sturm–Liouville problem · Oscillation theorem

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