A NUMERICAL APPROACH FOR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION WITH DELAY AND BOUNDARY LAYER

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ABSTRACT

In this study, we investigate the singularly perturbed convection diffusion problem for a class of Volterra integro-differential equation with delay. Differential equations including a small parameter in the highest order term are known as singularly perturbed differential equations. In general, the solutions of such equations exhibit multiscale phenomena. Within certain thin subregions of the domain, the scale of some derivatives is significantly larger than other derivatives. These thin regions of rapid change are called, boundary or interior layers, as appropriate. Such type of equations occur frequently in mathematical problems in the sciences and engineering for example, in fluid flow at high Reynold number, electrical networks, chemical reactions, control theory, the equations governing flow in porous media, the drift-diffusion equations of semi-conductor device physics, and other physical models [R4, R6]. On the other hand, Volterra delay integro-differential equations arise widely in mathematical models of biology, medicine, physics phenomena. In particular, the propagation of nervous impulse, population dynamics, polymeric liquids can be modelled by these equations [R3,R5].

Typically, the solutions of such problems include boundary or initial layer(s). The classical discretization methods to solve singularly perturbed problems do not work well and fail to give analytical solution when tends to zero. For this reason, it is necessary to develop suitable numerical methods that are uniformly convergent with respect to to solve this type of problems [R2,R4]. There are many studies in the literature using different numerical methods to solve first order delay Volterra integro-differential equations [R1]. However, we did not find any work for the second order delay Volterra integro-differential equations. This gap constitutes the motivation of this study.

To numerical approach for the problem, we construct a fitted difference scheme on a piecewise uniform mesh and analyze the error estimates. We prove that the method is almost first-order convergent with respect to the perturbation parameter in the discrete maximum norm. Moreover, we present the numerical experiments which support the theoretical results.

***K*eywords** Volterra delay integro-differential equation · Boundary layer · Finite difference method · Uniform convergence

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