ON DUAL BIQUATERNIONIC SEQUENCE INVOLVING VIETORIS' NUMBERS

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ABSTRACT

A sequence of rational numbers denoted by *Vietoris sequence*, appeard in a result published in [11], presented by L. Vietoris in 1958. The first Vietoris' numbers of the sequence are

$$1,\ \frac{1}{2},\ \frac{1}{2},\ \frac{3}{8},\ \frac{3}{8},\ \frac{5}{16},\ \frac{5}{16},\ \frac{35}{128},\ \frac{35}{128},\ \frac{63}{256},\ \frac{63}{256},\ \frac{231}{1024},\ \frac{231}{1024},\ \cdots,$$

which is related with the sequence A283208 in the OEIS in [9]. This sequence, which will be denoted by $\{v_n\}_{n\geqslant 0}$, is defined as

$$v_n = \frac{1}{2^n}\binom{n}{\lfloor \frac{n}{2} \rfloor}, \qquad n \geqslant 0,$$

and satisfies the following properties:

$$v_n = \begin{cases} 1, & n = 0 \\ \dfrac{n}{n+1}\, v_{n-1}, & n \text{ odd} \\ v_{n-1}, & n \text{ even} \end{cases}.$$

Many properties of Vietoris sequence can be seen in P. Catarino and R. de Almeida in [4] and I. Cação et al. in [2]. Special types of matrices that generates this number sequence were presented also in [4]. This sequence finds applications in harmonic analysis, demonstrated by the work of R. Askey and J. Steinig in [1]. It also plays a role in the theory of stable holomorphic functions, as presented in the paper by St. Ruscheweyh and L. Salinas [10].
In [3], we study some properties of the quaternionic sequence, $\{Q_n\}_{n\geqslant 0}$, with Vietoris' numbers as its components

$$Q_n = v_n + v_{n+1}\mathbf{i} + v_{n+2}\mathbf{j} + v_{n+3}\mathbf{k}, \quad n \in \mathbb{N}, \tag{1}$$

where $\{1, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$ is the standard basis in $\mathbb{R}^4$ satisfying the following multiplication rules:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1, \quad \mathbf{ij} = -\mathbf{ji} = \mathbf{k}, \quad \mathbf{jk} = -\mathbf{kj} = \mathbf{i}, \quad \mathbf{ki} = -\mathbf{ik} = \mathbf{j}.$$

To construct a dual biquaternionic sequence involving Vietoris's numbers, we start by considering the two-dimensional system of dual numbers defined by

$$\mathbf{D} = \{x + Iy : x, y \in \mathbb{R}, \quad I^2 = 0\}.$$

The conjugate of a dual number $z = x + Iy \in \mathbf{D}$ is defined by $\overline{z} = x - Iy$, and the modulus is defined as $|z| = \sqrt{z\overline{z}} = \sqrt{|x^2|} = |x|$. In [5] the basic properties of dual quaternions including fundamental operations, conjugates, inner product, vector product, and norm are discussed. Dual quaternions, are an extension of dual numbers, where the coefficients of real quaternions are dual numbers. It is well known several applications of dual quaternions, for instance, in [5] is mentioned the applications not only in animation, robotics, computer vision applications, theoretical kinematics, but also in the

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express and analyse of the physical properties of rigid bodies (see also the works [6, 7, 8] for more details). In this paper, using (1), we introduce an dual biquaternionic sequence, $\{V_n^p\}_{n\geqslant 0}$, given by

$$V_n^p = Q_n + IQ_{n+1}, \quad I^2 = 0, \quad n \in \mathbb{N},$$

and discusse some of its properties.

***K*eywords** Biquaternion · Dual numbers · Vietoris' number

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