ICMASE 2025

A COMPUTATIONAL METHOD FOR SOLVING COUPLED BURGERS' EQUATION

Neslişah İmamoğlu Karabaş[1,*], Gamze Tanoğlu[2]

[1] *Dept. of Mathematics, İzmir University of Economics*
[2] *Dept. of Mathematics, İzmir İnstitute of Technology*

ABSTRACT

The coupled viscous Burgers system plays a significant role in modeling a variety of phenomena in applied sciences, particularly in fluid dynamics and nonlinear acoustics. However, obtaining exact analytical solutions for such systems is often difficult and, in many cases, impractical. This study proposes a numerical algorithm based on a linearization technique to solve the coupled Burgers equations. The linearization is derived from the Newton–Raphson method. The presented scheme employs multiquadric radial basis functions (MQ-RBFs) for spatial discretization and the Crank–Nicolson method for time integration. By combining the meshless flexibility of radial basis functions with the stability of the Crank–Nicolson scheme, the method efficiently solves the coupled system. The accuracy and efficiency of the approach are demonstrated through the numerical solution of several benchmark problems from the literature. The novelty of this work lies in the integration of MQ-RBFs with Newton–Raphson-based linearization, enabling the efficient treatment of strong nonlinearities without requiring complex meshing. Moreover, the proposed methodology can be extended to a wide range of nonlinear partial differential equations and systems arising in heat transfer, traffic flow, and reactive transport modeling.

Keywords Linearization Technique · MQ-RBF · Crank-Nicolson · Coupled Burgers' Equation · Newton-Raphson Method

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Corresponding Author's E-mail: neslisah.karabas@ieu.edu.tr

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