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A BRIEF STUDY OF THE ONE PARAMETER LICHTENBERG NUMBERS

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ABSTRACT

In the 17th century, Georg Christoph Lichtenberg (1742–1799), as a result of his research on Chinese rings, introduced the following recurrence

$$Li_n = 2^n - 1 - Li_{n-1}\,, \tag{1}$$

for all integer $n \geq 1$, and with initial values $Li_0 = 0$ (see sequence $A000975$ in [8]). Hinz [5] and Cerda-Morales [3] called the sequence of numbers $\{Li_n\}_{n\geq o}$, the *Lichtenberg numbers* and established the nonhomogeneous recurrence relation

$$Li_n = Li_{n-1} + 2Li_{n-2} + 1\,, \tag{2}$$

for all integer $n \geq 2$, with initial terms $Li_0 = 0$ and $Li_1 = 1$. Equation (2) defines the sequence of *Ernst* numbers introduced by Soykan in cite soykan2022. For historical reasons, we will use the name Lichtenberg numbers, according to the authors Hinz [5], Stockmeyer [6], and also Heeffer and Hinz [7].

The Lichtenberg numbers are interesting because they are closely related to the well-known Jacobsthal numbers. The sequence of Jacobsthal numbers is denoted by $\{J_n\}_{n\geq 0}$ and defined by the recurrence relation $J_n = J_{n-1} + 2J_{n-2}$ with initial values $J_0 = 0$ and $J_1 = 1$ (see sequence $A001045$ in [8]). In fact, Cerda-Morales [3] determined the identity

$$Li_n = \frac{J_{n+2} - 1}{2}. \tag{3}$$

On the other hand, we highlight the work of Anatassov in [1, 2], who introduced a generalization of s-Jacobsthal numbers, as follows

$$J_{(s,n)} = \frac{s^n - (-1)^n}{s + 1}, \tag{4}$$

for every integer $n \geq 2$, arbitrary real number s, and initial values $J_{(s,0)} = 0$ and $J_{(s,1)} = 1$. In addition, we can define the s-Jacobsthal-Lucas numbers as follows

$$j_{(s,n)} = s^n + (-1)^n, \tag{5}$$

for $n \geq 2$, and with $j_{(s,0)} = 2$ and $j_{(s,1)} = 1$.

Motivated by identities (3) and (4), our goal is to introducethe s-Lichtenberg and s-Lichtenberg-Lucas numbers for some real s, and study some properties of this new sequence of numbers. More

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precisely, we give a recurrence for the s-Lichtenberg and s-Lichtenberg-Lucas numbers by using, respectively, the s-Jacobsthal and s-Jacobsthal-Lucas numbers. We show a relation between the s-Lichtenberg, s-Lichtenberg-Lucas, s-Jacobsthal and s-Jacobsthal-Lucas numbers and explore the connection between the s-Lichtenberg numbers, the Lichtenberg numbers, and the Jacobsthal numbers establishing some properties related to the s-Lichtenberg and s-Lichtenberg-Lucas numbers. In addition, Binet's formulas are obtained. Finally, we examine some properties of these new sequences, including the classical identities.

Keywords Lichtenberg numbers · Jacobsthal numbers · Binet's formula · Identities

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