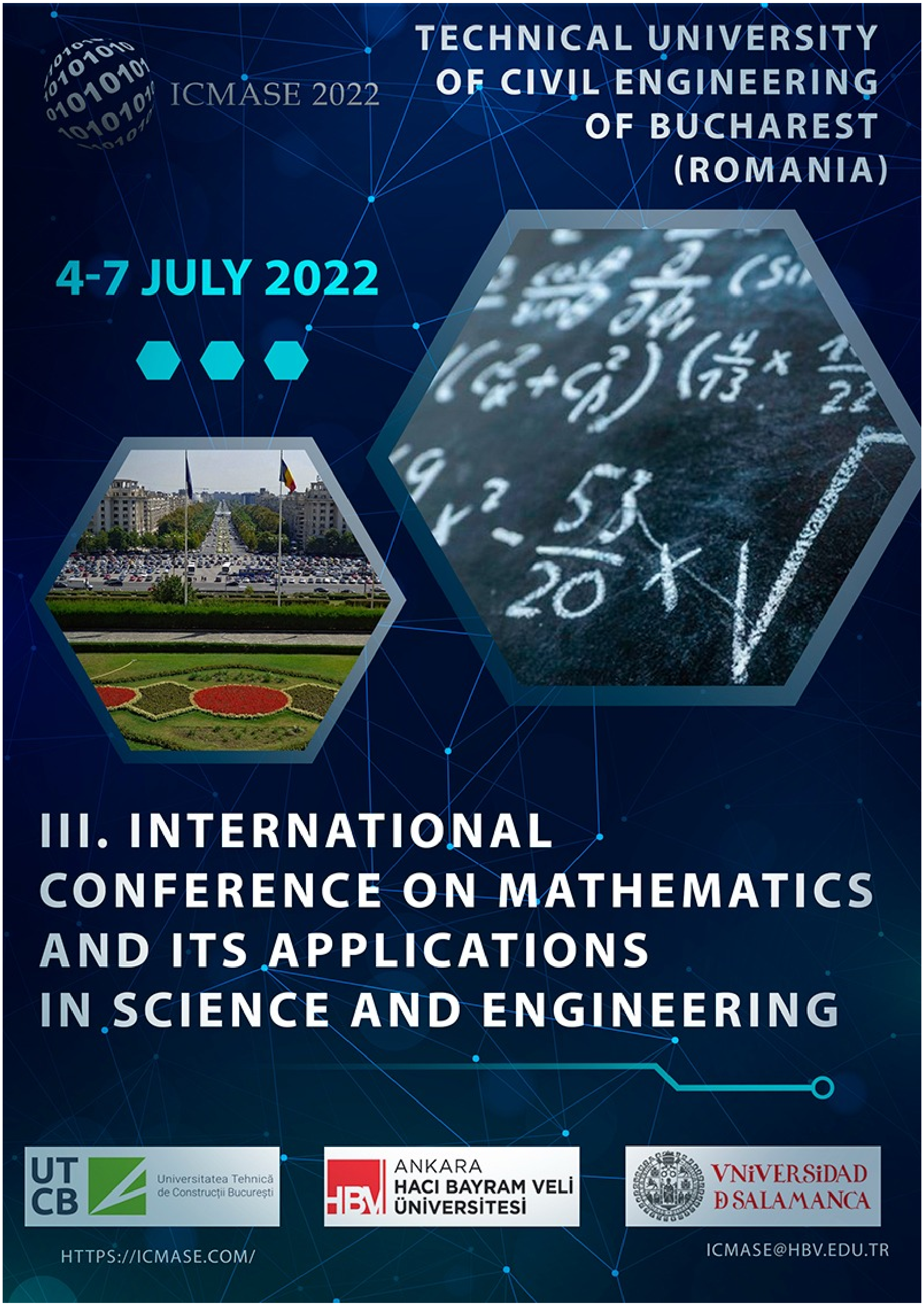
ICMASE 2022
TECHNICAL UNIVERSITY
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4-7 JULY 2022
III. INTERNATIONAL
CONFERENCE ON MATHEMATICS
AND ITS APPLICATIONS
IN SCIENCE AND ENGINEERING
UTCB Universitatea Tehnică de Construcţii Bucureşti
ANKARA HACI BAYRAM VELİ ÜNİVERSİTESİ
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Preface

This abstract booklet includes the abstracts of the papers that have been presented at III. International Conference on Mathematics and its Applications in Science and Engineering (ICMASE 2021) which is held in Technical University of Civil Engineering of Bucharest (Romania) between 4-7 July, 2022.

The aim of this conference is to exchange ideas, discuss developments in mathematics, develop collaborations and interact with professionals and researchers from all over the world about some of the following interesting topics: Functional Analysis, Approximation Theory, Real Analysis, Complex Analysis, Harmonic and non-Harmonic Analysis, Applied Analysis, Numerical Analysis, Geometry, Topology and Algebra, Modern Methods in Summability and Approximation, Operator Theory, Fixed Point Theory and Applications, Sequence Spaces and Matrix Transformation, Modern Methods in Summability and Approximation, Spectral Theory and Diferantial Operators, Boundary Value Problems, Ordinary and Partial Differential Equations, Discontinuous Differential Equations, Convex Analysis and its Applications, Optimization and its Application, Mathematics Education, Applications on Variable Exponent Lebesgue Spaces, Applications on Differential Equations and Partial Differential Equations, Fourier Analysis, Wavelet and Harmonic Analysis Methods in Function Spaces, Applications on Computer Engineering, Flow Dynamics. However, the talks are not restricted to these subjects.

Thanks to all committee members.

We wish everyone a fruitful conference and pleasant memories from ICMASE 2022.

Ion Mierlus-Mazilu

Chair, ICMASE 2022

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KASTRIOT ZOTO, ILIR VARDHAMI

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De Moivre's Formulas for Split Octonions

Mücahit AKBIYIK[1]

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Abstract

In this talk, we calculate De Moivre's formulas of split octonions. We examine the $n^{th}-$ roots of split octonions. Also, we give an illustrative example. In additon to this, we present polars form for two type of split octionions.

***K*eywords** Split Octonions; Matrix representation; De Moivre's formula.

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SOBOLEV ORTHOGONALITY OF A CLASS OF FINITE ORTHOGONAL POLYNOMIALS

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ABSTRACT

In this paper, a class of the finite Sobolev orthogonal polynomials is considered. The aim is to set an orthogonal structure in the Sobolev space and investigate the structure for negative parameters in some special cases.

***K*eywords** Sobolev space · finite orthogonal polynomials · partial differential operators

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Derivative-Free Finite-Difference Homeier Method for Systems of Nonlinear Models

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Abstract

An efficient derivative free method for finding roots of nonlinear equations was implemented in this paper. The third-order Homeier's method has been taken as the basis for this work, which can be derived by using Newton's theorem for the inverse function and derive a new class of cubically convergent Newton-type methods. Several nonlinear problems, including nonlinear equations, complex equations, and nonlinear systems of equations, have been taken to compare the efficiency of the proposed method to other popular derivative-free schemes. Results show that the proposed method outperformed the considered published methods. The presented scheme needs fewer iterations to achieve the desired solution, with an order of convergence of about 2.5, which is higher than the convergence order of the compared methods, and one of the popular nonlinear equation solvers used to compare with our proposed method is secant method with order of convergence 1.618 in the absence of derivative. When using the proposed method for solving systems of nonlinear equations, the Jacobian problem can be avoided by following the procedure in Broyden's method. Thus, the proposed method can be considered as an uppermost method giving faster convergence to find the roots of nonlinear equations in the absence of the derivative for uni-variate nonlinear equations with complex roots as well as for the multivariate systems of nonlinear equations. We expect that our proposed method will undoubtedly be very useful and effective to the scientific and industrial community.

***K*eywords** Homeier method · Secant method · Nonlinear equations · Derivative-free methods · Iterative methods · Broyden's method · Order of convergence

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A SPECTRAL COLLOCATION METHOD WITH CONVERGENCE ANALYSIS FOR SOLVING NONLINEAR FRACTIONAL FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT

In this article, an efficient and accurate spectral numerical method is presented for solving nonlinear fractional Fredholm integro-differential equations (FFIDEs) with the initial condition. The proposed method is based on the shifted Legendre-Gauss-Lobatto collocation (SL-GL-C) method for fractional derivative, described in the Caputo derivative sense. We adapt the SL-GL-C algorithm to solve the nonlinear FFIDEs. Moreover, we provide a framework for studying the rate of convergence of the proposed algorithm. The effectiveness and validity of the method have been proved by solving four numerical examples. Besides, we give a numerical test example to show that the approach can preserve the non-smooth solution of the underlying problems.

QUATERNION ALGEBRAS AND THE ROLE OF QUADRATIC FORMS IN THEIR STUDY

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ABSTRACT

For a long time, Hamilton tried to model a 3-dimensional space with a structure similar to that of complex numbers, whose addition and multiplication are found in bidimensional space. In this sense, Hamilton realized that it would need a fourth dimension and thus invented the term of *quaternions* for real space, generated by the elements $1, i, j, k$, relative to multiplication. (John Voight, *Quaternion algebras*, March 27, 2021, p.1)

But further, Dickson was the first which considered quaternion algebras over an arbitrary field. He started it by generalizing those algebras in which each element satisfies a quadratic equation, he set out a diagonalizable basis for such an algebra and analyzed the conditions to be a division algebra. (John Voight, *Quaternion algebras*, March 27, 2021, p.9). This led him to what he afterwards called the *generalized quaternion algebra*, for which:

$$i^2 = \alpha, j^2 = \beta, k^2 = -\alpha\beta$$

$$ij = -ji = k, ik = -ki = \alpha j, kj = -jk = \beta i$$

One way to classify and also, characterize quaternion algebras is by using quadratic forms. In this sense, we will recall some basic notions related to the theory of quadratic forms and we will emphasize their connection with quaternion algebras. We will not include the proofs, but they can be found in any of the references mentioned.

***K*eywords** Bilinear forms · Quadratic forms · Quaternion algebras · Norm form · Pure quaternions

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THE MOORE-PENROSE INVERSE IN RICKART $*$-RINGS

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ABSTRACT

The main purpose of this paper is to introduce several new necessary and sufficient conditions for the existence of the Moore-Penrose inverse of an element in a ring R are obtained. In addition, the formulae of the Moore-Penrose inverse of an element in a ring are presented.

***K*eywords** The Moore-Penrose inverse · $*$-ring · A Rickart $*$-ring · Projection element · Weak-supported element

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ERROR DETECTION AND CORRECTION FOR CODING THEORY ON K-ORDER GAUSSIAN FIBONACCI MATRICES

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ABSTRACT

In this study, we consider the coding theory for Gaussian Fibonacci numbers of order-k. This coding method is based on the Q_k, R_k and $E_n^{(k)}$ matrices. In this respect, it differs from classical encryption methods. Unlike classical algebraic coding methods, this method theoretically allows for the correction of matrix elements that can be infinite integers. Error detection criterion is examined for the case of $k = 2$ and this method is generalized to k and error correction method is given. In the simplest case, for $k = 2$, the correct capability of the method is essentially equal to $93,33\%$, exceeding all well known correction codes. It appears that for a sufficiently large value of k, the probability of decoding error is almost zero.

Keywords Gaussian Fibonacci numbers · k-order Gaussian Fibonacci numbers · k-order Gaussian Fibonacci matrices · k-order Gaussian Fibonacci · Error Detection, Correction coding/decoding.

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FUZZY CLUSTERING BASED FUZZY REGRESSION FUNCTION IN Z-ENVIRONMENT

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ABSTRACT

Real-life applications involve various epistemic uncertainties. These uncertainties arise from the limitations in data or measurement, and modeling approximations. As known, fuzzy theory (Zadeh, 1965) deals with all these uncertainty resources. Besides, fuzzy set theory has been developed to analyze and reduce its own uncertainty due to the precise nature of primary membership. There have been proposed numerous versions of type-1 fuzzy sets, such as interval or general type-2 fuzzy sets, intuitionistic fuzzy sets, neutrosophic fuzzy sets, Z-numbers, etc. One of the remarkable proposals is Z-numbers to improve perception-based decisions. Zadeh (2011) introduced Z-numbers to measure the reliability of information in decision-making. A Z-number is an ordered pair fuzzy number, (A,R). The first component, A, is a restriction on a random variable, and the second, R, is the reliability of the first one. Hybrid methods including Z-numbers are needed to make reliable information-based decisions. The most important issue in decision problems is to model any given system by identifying the relationships between its components (input-output variables). Türkşen (2008) proposed fuzzy regression function as a fundamental of system modeling in fuzzy environment. The original inputs and the memberships of any given system are used as new inputs in Türkşen's fuzzy regression models. Thus, an effective model structure is created for the system and its uncertainty. There are limited studies regarding the regression models in Z-environment (i.e. Zeinalova et.al., 2017; Ezadi and Allahviranloo, 2017; Poleshchuk, 2020). Fuzzy system modeling based on Z-numbers can handle discrepancies in judgments by defining functional relationships between linguistic input-output variables. Thus, in this study, we propose a Z-number valued fuzzy regression function with fuzzy clustering. Fuzzy c-means algorithm (Bezdek, 1981) is used in order to form a mathematical model using initial Z-numbered input-output variables and their memberships. The performance of the proposed approach is

examined using an illustrative example related to the maturity level of digital transformation.

Keywords Z-numbers · Fuzzy clustering · Fuzzy regression function

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An Individual Work Plan to Influence Educational Learning Paths in Engineering Undergraduate Students

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Abstract

Issues related to the failure of mathematics in the engineering education and the negative impact of these difficulties in the success of the Differential and Integral Calculus curricular units taught in engineering courses are a problem to which we have devoted our attention and research [1]. Most students entering higher education have a weak mathematics preparation. It's even more aggravated due to the different areas of knowledge of its experience when entering engineering courses.

The Support Center for Mathematics in Engineering (CeAMatE) in Coimbra Engineering Institute is a space dedicated to accompanying students to overcome difficulties in basic and elementary Mathematics knowledge, which is essential for full integration into engineering courses. In this Center, there is a set of activities and resources based in Primary and Secondary Education programs in Portugal, and in the Core Zero outcomes from the SEFI guidelines, Mathematics for the European Engineer – A Curriculum for the Twenty-First Century [2]. These outcomes are grouped taking into account the different topics from Algebra, Analysis and Calculus, Discrete Mathematics, Geometry and Trigonometry, and Statistics and Probability. The Center also incorporates an e-learning component, adapting to learning styles and students' knowledge levels.

Diagnostic test results provide information about the mathematical contents that should be worked with students. These results make possible to define individual working plans that allow autonomous work in overcoming the difficulties in mathematics [3, 4]. Moreover, these individual plans will describe the evolution of students' learning, through the monitoring and reformulation of knowledge acquisition.

The objective of this study is to understand the relationship between the Diagnostic Test results, the attendance at CeAMATE and the evaluations obtained in the Curricular Unit of Differential and Integral Calculus.

***K*eywords** Mathematics Knowledge · Diagnosis Test · Engineering · Differential and Integral Calculus · Individual Working Plan

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A SEQUENTIAL CONSTRUCTION OF HYPERCOMPLEX ALGEBRAS

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ABSTRACT

The structure of algebra over a vector space is a way of obtaining an extension of sets of numbers.Entering complex-split numbers (doubled numbers) is a simple way to get an extension of real numbers. This has the square of the imaginary element 1.Similar to the introduction of quaternions, octonions and sedenions, the structure of complex-split numbers can be doubled in size. A series of commutative algebras is obtained. The multiplication table of the basis elements for these algebras can be easily generated using a binary write of the index of the basis elements. These algebras can be generalized by introducing parameters in defining the squares of the imaginary elements of the base similar to generalized quaternions and octonions.Thus, the introduction of hypercomplex algebras into the known form can be achieved.Thus, the study of the simplest way to introduce hypercomplex algebras is proposed. The properties of the introduced algebras and the objectives of their introduction are discussed. These refer to the trace and norm functions, given by the addition of a hypercomplex number and its conjugate.For the introduced algebras the norm function is different from the square of the Euclidean norm on the considered linear space. It is investigated how to perform operations in a non-associative algebra. It is shown that in this there must be only one way of associating the factors.

Keywords complex-split numbers · hypercomplex algebras · binary index · bitwise XOR · generalized quaternions

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ICMASE 2022

Stability Analysis Of a Caputo Type Fractional Waterborne Infectious Disease Model

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Abstract

In this presentation we propose a Caputo type fractional waterborne bacterial infection model with saturation effect of infectious population on the transmission of disease caused by waterborne bacteria. The total population size $N(t)$ is divided into two compartments: susceptible individual $S(t)$ and infectious with symptoms $I(t)$ at time $t \geq 0$. Furthermore, we consider a compartment $B(t)$ that reflects the bacterial concentration at time t. We assume positive natural death rate μ. Susceptible individuals can become infected by contact with infected individuals at rate $\frac{\rho I(t)}{1+m_1 I(t)}$. Susceptible individuals can become infected with waterborne disease, like cholera by contact with contaminated sources at rate $\frac{\beta B(t)}{1+m_2 I(t)}$, where $\beta > 0$ is ingestion rate of the bacteria through contaminated sources. We assume nonnegative death rate δ caused by infection. Each infected individual contributes to the increase of the bacterial concentration at rate ξ. On the other hand, the bacterial concentration can decrease at mortality γ. With these assumptions we have the following Caputo type fractional waterborne disease model

$$\begin{cases} D^\alpha S(t) = \Lambda - \left(\frac{\rho I(t)}{1+m_1 I(t)} + \frac{\beta B(t)}{1+m_2 I(t)}\right) S(t) - \mu S(t), \\ D^\alpha I(t) = \left(\frac{\rho I(t)}{1+m_1 I(t)} + \frac{\beta B(t)}{1+m_2 I(t)}\right) S(t) - \delta I(t) - \mu I(t) \\ D^\alpha B(t) = \xi I(t) - \gamma B(t). \end{cases} \tag{1}$$

For this model with initial conditions

$$S(0) \geq 0, \quad I(0) \geq 0, \quad B(0) \geq 0, \tag{2}$$

we first find existence and uniqueness of the solution of system (1) with initial conditions (2) which remains in a positively invariant region

$$\Omega = \left\{ (S,\, I,\, B) \in \mathbb{R}^3_+ \ :\ S + I \leq \frac{\Lambda}{\mu}, \quad 0 \leq B \leq \frac{\xi\Lambda}{\mu\gamma} \right\}. \tag{3}$$

Secondly, we find the existence of disease-free equilibrium point $E_0 = (\Lambda/\mu,\, 0,\, 0)$ and endemic epidemic equilibrium point $E_* = (S_*,\, I_*,\, B_*)$ and define basic reproduction number R_0 of this system. Next, we investigate the local stabilities of equilibriums E_0 and E_* by using Jacobian matrix of system (1) at these equilibriums and global stability of the disease-free equilibrium using Lyapunov's method.

***K*eywords** Fractional calculus · Caputo derivatives · Epidemiology · Equilibrium · Stability

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A NOTE ON k-TELEPHONE AND INCOMPLETE k-TELEPHONE NUMBERS

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ABSTRACT

The telephone numbers, also known as involution numbers, are given by

$$T_n = T_{n-1} + (n-1)T_{n-2},\ n \geq 2 \qquad (4)$$

with initial terms $T_0 = T_1 = 1$. The recurrence relation (4) of the sequence $\{T_n\}_n$ was found by Heinrich August Rothe in 1800 ([5]) when counting the involutions (that is, permutations that are their own inverse) in a set of n elements.

This sequence can also be seen as the number of possible patterns of connections between the n subscribers of a telephone service, therefore the designation *telephone numbers*. Another application of the telephone numbers is to graph theory, with T_n given by the number of matchings (Hosoya index) in a complete graph. In recreational mathematics, the nth telephone number T_n is the number of ways to place n rooks on an $n \times n$ chessboard such that no two rooks attack each other and such that the configuration of the rooks is symmetric under a diagonal reflection of the board.

In this work, the k-telephone and the incomplete telephone numbers are introduced using the same methodology that was applied to other sequences, such as the Fibonacci sequence ([3, 4, 6]), the Pell sequence ([5, 6, 7, 8]) or the Leonardo sequence ([8]) .

Similarly to the works with the Fibonacci sequence, for any positive real number k, the k-telephone sequence $\{T_{k,n}\}_{n\in\mathbb{N}}$ is defined by

$$T_{k,0} = 1, \quad T_{k,1} = k, \quad T_{k,n} = kT_{k,n-1} + (n-1)T_{k,n-2}$$

and it is proved that the explicit formula for this sequence is given by

$$T_{k,n} = \sum_{i=0}^{\lfloor n/2 \rfloor} \frac{n!}{2^i(n-2i)!i!} k^{n-2i}.$$

Furthermore, the incomplete telephone numbers are defined by

$$T_n^l = \sum_{i=0}^{l} \frac{n!}{2^i(n-2i)!i!} \qquad 0 \leq l \leq \left\lfloor \frac{n}{2} \right\rfloor$$

and the incomplete k-telephone numbers are defined by

$$T_{k,n}^l = \sum_{i=0}^{l} \frac{n!}{2^i(n-2i)!i!} k^{n-2i}, \qquad 0 \leq l \leq \left\lfloor \frac{n}{2} \right\rfloor .$$

The recurrence relations and relevant properties for the k-telephone numbers and the incomplete k-telephone numbers are presented in this work.

***K*eywords** Telephone numbers · k-telephone numbers · Incomplete telephone numbers · Incomplete k-telephone numbers

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A Note on Special Matrices involving k-Bronze Fibonacci Numbers

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Abstract

Numerical sequences are a source of very interesting and challenging mathematical problems and have attracted the attention of many researchers. Many developments have been done concerning the well-known Fibonacci sequence or the Lucas sequence, but also many works have emerged on other sequences of numbers, polynomials, quaternions, octonions, sedenions, etc. We refer, for example, to the works on Hybrid numbers [4, 12], applications of Fibonacci and Lucas numbers [13], Leonardo's numbers [6], k-Pell generalized numbers of order m, where m is a non-negative integer [7], Gaussian Fibonacci sequences [3], Gaussian Lucas sequences [10], Gaussian Pell sequences and Gaussian Pell-Lucas sequences [9], Gaussian Jacobsthal sequences [2], and Gaussian Bronze Fibonacci sequences [11]. Newer developments appeared recently concerning third-order Bronze Fibonacci numbers [1], on Vietoris'numbers sequence [8], on generalized sequences of numbers known as k-Fibonacci numbers, k-Jacobsthal numbers, k-Pell numbers, balancing numbers, k-telephone numbers, hyper k-pell numbers and incomplete numbers [5].

In 1985, Levesque [3] deduced an important formula, known as the Binet formula, which is obtained in terms of the roots of the characteristic equation associated with the recurrence relation defining the considered sequence. Binet's formula allows one to find the general term of a sequence, without having to resort to other terms of the sequence, thus being an important tool to study some properties of the sequence.

In the present work, we take as our starting point the Bronze Fibonacci sequence, $\{BF_n\}_{n\geq 0}$, listed in the Online Encyclopedia of integers sequences [15] as the sequence A006190, and defined by the following recurrence relation

$$BF_{n+2} = 3BF_{n+1} + BF_n, \tag{5}$$

with the initial conditions $BF_0 = 0$ and $BF_1 = 1$. We consider a generalization of Bronze Fibonacci sequence, in which the recurrence formula depend on one parameter k. We call this new sequence, the k-Bronze Fibonacci sequence, denoted by $\{BF_{k,n}\}_{n\geq 0}$, and defined by the following recurrence relation

$$BF_{k,n+2} = 3BF_{k,n+1} + kBF_{k,n}, \quad k \in \mathbb{Z}^+, \tag{6}$$

with the initial conditions $BF_{k,0} = 0$ and $BF_{k,1} = 1$,
Our goal is to give alternative ways to determine the general term of the k-Bronze Fibonacci sequence involving some special tridiagonal matrices and their determinants.

***K*eywords** Bronze Fibonacci numbers · Tridiagonal matrices · general term

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On the Statistical Properties of the Deformed Algebras on the Jackson q-Derivative

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Abstract

The property of Tsallis q-entropy is introduced. The q-deformed logarithm produced by Tsallis q-entropy is examined. In this study, this logarithm is extended to the correlation coefficient type. Since a new deformed difference based on the statistical properties such as correlation coefficient and the M-estimation method, a new derivative and its corresponding integral can be proposed. The simulation study is performed to observe the results of the proposed algebras.
***K*eywords** Algebras · Statistical inference · Tsallis entropy · q-calculus

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MULTICOVARIANCE AND MULTICORRELATION FOR p-VARIABLES

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ABSTRACT

The covariance and correlation are important indicators to measure the linear dependence between two variables. The multivariables as p-variables in the research are commonly observed in the engineering, social science, medical examination, etc. when p-variables has a negative and positive dependence. In this study, The M-function in the M-estimation method is used to define a multivariate generalization for the correlation. A linear dependence among p-variables are generated by use of the artificial data sets which have a linear dependence and normal distribution. The different sample sizes and different number of p-variables are used while the simulation study is performed. Even if the number of p-variables is greater than the number of sample size, the dependence coefficient shows better performance. Thus, the numerical results have shown that the proposed multicorrelation coefficient show better performance to detect the value of degree of dependence among p-variables.
***K*eywords** Correlation · M-estimation · Multicorrelation · Dependence

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ICMASE 2022

Experience in Teaching Mathematics to Engineers: Students v.s Teacher Vision

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Abstract

Classroom experience is very important for developing the skills necessary for effective teaching. Even when educational success is defined by test scores, research shows that teachers' effectiveness increases with experience and, under favourable circumstances, continues to increase throughout their entire career. Many experiments have been made in the application of new teaching/learning methodologies in mathematics. In higher education, more specifically in the teaching of mathematics in engineering, these experiences are quite scarce. Collaborative learning (CL) allows for meaningful learning in which the student takes an active role in their teaching/learning process, developing a wide range of skills [1]. The CL experience was carried out in the Calculus I course, with 25 Electrical Engineering students, on academic year 2021/2022. Differential and Integral Calculus to calculate areas and volumes of solids of revolution was the chosen theme, using GeoGebra to dynamically model 3D objects and GeoGebra AR to visualize them directly in augmented reality [2]. The experience had the participation of 6 groups of students and lasted 3 hours. It was necessary to reflect, think and plan the class so that it could play an important role in improving the quality of teaching and learning at this academic level. The class was administered by the teacher as a guide, observing and writing down some information [3]. The students, in addition to developing the proposed activity, answered an initial and final quiz. This paper intends to present the opinions and reflections of the students and the teacher about the effect of their participation in a CL experience. The final observations suggest that CL is an important methodology in the context of training, with a view to reflection and improvement of pedagogical practice. As such, it should be included, whenever possible, in the planning of mathematics lessons for engineering and for higher education in general.

Keywords Teaching and Learning experiences · Mathematics for engineering · Collaborative learning · Exploratory approach

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THE EFFECT (IMPACT) OF PROJECT-BASED LEARNING THROUGH AUGMENTED REALITY ON HIGHER MATH CLASSES

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ABSTRACT

Mathematics is essential in the training of engineers. A weak mastery in this area can affect other course subjects that require math skills. The lack of motivation that students feel in relation to this subject influences their poor academic performance [1]. It is necessary that the teaching of Mathematics be more creative and stimulating, considering modern society and the interests of students. Project-based learning can help to increase engagement, improve student interaction, and promote student success in Mathematics. In this paper, project-based learning and its study are presented to qualitatively assess students' motivation and performance in this type of teaching methodologies. During a 3-hour class of the second semester of Calculus I in Electrical Engineering degree, students were proposed to develop a project using augmented reality. Different views, examples, and applications for 3D object modelling as well as area and volume calculation using integral calculation were implemented [2]. Augmented reality was then exposed focusing on the competencies that are considered in the learning project. To develop the referred experience, the interactions established between students and student-teacher were analysed through direct observation [3], the analysis of documents delivered by the students, the satisfaction questionnaires carried out and the analysis of the evaluation grids. The analysis of the collected data allowed us to conclude that the experience proposed in a classroom context, with the aim of motivating students to learn Mathematics, proved to be effective and productive.

Keywords Project-based learning · Mathematics for engineering · Students' motivation · Students' performance

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Some Spectral Properties of Operators Generated by Quantum Difference Equations

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Abstract

In this talk, we first get the q–analogous of the Sturm–Liouville equation which can be written as

$$D_q\left[p\left(\frac{t}{q}\right)D_q y\left(\frac{t}{q}\right)\right]+r(t)y(t)=0,\quad t\in q^{\mathbb{Z}}, \tag{7}$$

where the functions p and r are defined on $q^{\mathbb{Z}}$ with $p(t)\neq 0$ for all $t\in q^{\mathbb{Z}}$ [6,7]. Equation (1) is called the second order selfadjoint linear homogenous q–difference equation. After making required calculations using q-derivative, it can be seen that (1) turns out to be

$$q\gamma(t)y(qt)+\beta(t)y(t)+\gamma\left(\frac{t}{q}\right)y\left(\frac{t}{q}\right)=0,\quad t\in q^{\mathbb{Z}}, \tag{8}$$

then we consider the q–difference expression

$$q\gamma(t)y(qt)+\beta(t)y(t)+\gamma\left(\frac{t}{q}\right)y\left(\frac{t}{q}\right)=\lambda y(t),\quad t\in q^{\mathbb{Z}}, \tag{9}$$

which was intensively studied in [1,2,3,4], where $\gamma(t)\neq 0$ for all $t\in q^{\mathbb{Z}}$. The spectral parameter λ can be chosen exponential function as $\lambda := 2\sqrt{q}\cos z$ or polynomial function as $\lambda := \sqrt{q}\,(z+z^{-1})$. Over the years, Equation (3) has been handled in special case under some impulsive conditions and investigated in [5].

The main aim of this talk is to investigate some spectral properties of Equation (3) such as getting the analytic properties and asymptotic behaviours of the Jost solution and obtain the continuous spectrum of the related operator.

Keywords q–difference equation · q–difference operator · Jost solution · eigenvalue · spectral singularity · asymptotic

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The Theory of Dirac Equations and System of Difference Equations

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Abstract

Spectral analysis, a sub–branch of applied mathematics and functional analysis, examines the solutions of boundary and initial value problems using "operator theory". Operator theory is no doubt a diverse area that has grown out of linear algebra and complex analysis, and is often described as the branch of functional analysis that deals with bounded linear operators and their spectral properties. In mathematical models, an operator is determined compatible with the differential equation used and the properties of the operator are investigated. Differential operators generated by the differential equations are the first operator type handled in the literature. In particular, Sturm–Liouville operator, known as one dimensional Schrödinger operator is the most widely studied operator having a tremendous potential for applications in quantum theory.

Since the beginning of 18th. century, various investigations have been done by various authors, but with the pioneering of Naimark. In his first study, Naimark considered the Sturm–Liouville operator generated by the Sturm–Liouville equation

$$-y'' + q(x)y = \lambda y, \quad x \in \mathbb{R}_+ := [0, \infty) \tag{10}$$

and the initial condition

$$y'(0) - hy(0) = 0, \tag{11}$$

where q is a complex potential and λ is a spectral parameter [6]. Also, the modellings of certain problems in engineering, physics, control theory and quantum mechanics and other areas have led to a rapid development of the theory of Dirac equations and discrete Dirac equations.

In this presentation, at first, we introduce the matrix equation

$$K\frac{d\psi}{dx} + P(x)\psi = \mu\psi, \quad \psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}, \quad x \in [0,\pi], \tag{12}$$

where

$$K = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \; P(x) = \begin{pmatrix} P_{11}(x) & P_{12}(x) \\ P_{21}(x) & P_{22}(x) \end{pmatrix}, \; P_{12}(x) = P_{21}(x),$$

P_{ij} are real valued functions which are continuous on the interval $[0,\pi]$ for $i,j = 1,2$ and μ is a spectral parameter. It follows from (3) that

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \psi_1'(x) \\ \psi_2'(x) \end{pmatrix} + \begin{pmatrix} P_{11}(x) & P_{12}(x) \\ P_{21}(x) & P_{22}(x) \end{pmatrix} \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} = \mu \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}.$$

Using the last equality, we have

$$\begin{pmatrix} \psi_2'(x) \\ -\psi_1'(x) \end{pmatrix} + \begin{pmatrix} P_{11}(x)\psi_1(x) + P_{12}(x)\psi_2(x) \\ P_{21}(x)\psi_1(x) + P_{22}(x)\psi_2(x) \end{pmatrix} = \begin{pmatrix} \mu\psi_1(x) \\ \mu\psi_2(x) \end{pmatrix}.$$

Then, we arrive at that the Equation (3) is equivalent to the system of two simultaneous first order ordinary differential equations

$$\begin{cases} \psi_2' + P_{11}(x)\psi_1 + P_{12}(x)\psi_2 = \mu\psi_1, \\ -\psi_1' + P_{21}(x)\psi_1 + P_{22}(x)\psi_2 = \mu\psi_2. \end{cases} \tag{13}$$

In the case that $P_{12}(x) = P_{21}(x) = 0,\; P_{11}(x) = s(x) + m,\; P_{22}(x) = s(x) - m$, where s is a potential function and m is the mass of a particle, the system (4) is called a *one dimensional stationary Dirac system* in relativistic quantum theory [5]. Many studies exist about the spectral theory of Equation (4) [4,5,7]. Over the years, similar spectral properties have been obtained for the discrete case of Dirac equations and extensively studied in [1-3].

***K*eywords** Dirac equation · Discrete Dirac equation · Jost solution · Wronskian · eigenvalue

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ICMASE 2022

Renewed Neutrosophic Soft Graphs with Some New Operations

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Abstract

It is celarly state that uncertainty arises from various areas cannot be captured within a single mathematical approach. Mathematical modelling of problems with uncertainty and development of solutions accordingly is one of the most important issue in interdisciplinary research. For this reason, many theory have been developed for solving problems involving uncertainty. Some of these are fuzzy set theory, intuitionistic fuzzy set theory and fuzzy soft set theory. On the other hand, the neutrosophic set is a new mathematical approach which is developed for dealing with incomplete and indeterminate information. Neutrosophic set is a generalization of the intuitionistic fuzzy set theory. The neutrosophic sets are expressed with the help of three membership functions named truth, indeterminacy and falsity membership function. As compared to the other fuzzy models, the neutrosophic soft models provide more sensitive evaluation for the complex systems. Graph theory which is used to solve the complicated problems in many different fields is an important mathematical tool. Graphs are used to put forth a relationship between elements in a given set. Graph theory and fuzzy graph theory are finding an many number of applications in modeling complicated systems because of its provide conveniences. Theoretical point of view, graph structures have been many times evaluated on different sets especially soft sets, fuzzy soft sets, neutrosophic sets, neutrosophic soft sets etc. This study is designed with the renewed concept of neutrosophic soft graph structures which is a combination of graphs and neutrosophic soft sets. We redefine notions of neutrosophic soft graphs and neutrosophic soft subgraphs from a different perspective taking into account the shortcomings of previous studies. Also, we introduce some new operations on neutrosophic soft graphs and elaborate them with suitable examples by using neutrosophic soft sets. Moreover, we investigate some remarkable properties of neutrosophic soft graphs via concepts given.

***K*eywords** Neutrosophic soft set · Graph · Neutrosophic soft graph

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ICMASE 2022

A GENERALIZATION OF MULTIPLE ZETA VALUES

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ABSTRACT

This study consists of two parts: Part 1: Recurrent Sums [23] and Part 2: Multiple Sums [24]. A combined abstract for both is as below:

Multiple zeta star values (MZSV) and Multiple zeta values (MZV) have become of great interest due to their numerous applications in mathematics and physics. In this study, we propose a generalization of MZSVs and MZVs, which we will refer to as *recurrent sums* and *multiple sums*, where the reciprocals are replaced by arbitrary sequences. We introduce a toolbox of formulas for the manipulation of such sums. We begin by developing variation formulas that express the variation of such sums in terms of lower order recurrent/multiple sums. We then proceed to derive theorems (which we will call inversion formulas) which show how to interchange the order of summation in a multitude of ways. Additionally, we derive a set of partition identities that we use to prove a reduction theorem that expresses such sums as a combination of simple non-recurrent sums. We use these theorems to derive new results for multiple zeta (star) values and recurrent/multiple sums of powers. Later, we present a variety of applications including applications concerning polynomials and MZVs such as generating functions and expressions for $\zeta(\{2p\}_m)$ and $\zeta^\star(\{2p\}_m)$. Finally, we establish the connection between multiple sums and recurrent sums. By exploiting this connection, we provide additional partition identities for odd and even partitions.

***K*eywords** Recurrent sums, Multiple sums, Partitions, Multiple zeta star values, Multiple zeta values, Riemann zeta function, Viète's formula, Polynomials, Generating function, Faulhaber formula.

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Some Results for Matrix Sturm-Liouville Equations with a Point Interaction

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Abstract

Spectral theory of differential equations dates back to 1960 [6] and intensively studied in [5, 3, 4]. In [5], the author investigated the Sturm–Liouville boundary value problem

$$\begin{cases} -y'' + q(x)y = \lambda^2 y, \quad 0 \le x < \infty \\ y(0) = 0, \end{cases} \tag{14}$$

under the condition

$$\int_0^\infty x\,|q(x)|\,dx < \infty,$$

where λ is a spectral parameter, q is a complex valued function. Moreover, Marchenko gets a bounded solution of (14) satisfying the condition

$$\lim_{x\to\infty} y(x)e^{i\lambda x} = 1, \quad \lambda \in \overline{\mathbb{C}}_+ := \{\lambda \in \mathbb{C},\ \operatorname{Im}\lambda \ge 0\}, \tag{15}$$

which is called Jost solution of (14). Jost solutions play an important role in spectral analysis of differential and difference equations. Recently, matrix valued equations have been studied in different cases by various authors [1, 2].

The main purpose of this talk is to give some spectral results for the matrix analogous of the Sturm-Liouville equation (1) under the effect of an impulsive condition, i.e., point interaction [7]. Thus, in this presentation, we will consider the impulsive matrix Sturm-Liouville operator L acting in the Hilbert space $\mathcal{L}_2(\mathbb{R}_+, S)$ generated by

$$\begin{aligned} -Y'' + Q(x)Y = \lambda^2 Y, \quad x \in [0,1) \cup (1,\infty) \\ Y(0) = 0 \end{aligned} \tag{16}$$

with the point interaction

$$\begin{aligned} Y(1^+) = C_1 Y(1^-) \\ Y'(1^+) = C_2 Y'(1^-), \end{aligned} \tag{17}$$

where λ is a spectral parameter, $Q = [q_{ij}]_{nxn}$ is a matrix valued function such that $Q = Q^*$ satisfying

$$\int_0^\infty x\|Q(x)\|dx < \infty. \tag{18}$$

Throughout this study, we assume that C_1 and C_2 are self– adjoint diagonal matrices such that all eigenvalues of C_1 and C_2 are positive.

Note that, (17) is a transmission condition or point interaction for the Equation (16) and $x = 1$ is the transmission point for the boundary value problem (16)-(17). After a brief introduction to the classical matrix valued differential equations, the solutions of corresponding equation will be investigated. By the help of the asymptotic properties and the Jost function, the eigenvalues and spectral singularities of the corresponding operator will be discussed.

***K*eywords** Sturm-Liouville equation · Point interaction · Jost solution · Spectral theory

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WEYL THEORY FOR THE FRACTIONAL STURM-LIOUVILLE EQUATIONS

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ABSTRACT

The fundamental results have been obtained in 1910 by Weyl [5] on the following second order Sturm–Liouville equation

$$-\left(p(x)y'\right)' + q(x)y = \lambda y,\ x \in [0,\infty), \tag{19}$$

where p, q are real valued functions, $p > 0$ and p^{-1}, q are locally integrable functions on the given interval. These results are about the number of the square integrable solutions of the Equation (1). In fact, according to the results of Weyl, at least one solution of the two linearly independent solutions of Equation (1) must lie in the Lebesgue space consisting of all functions whose square is integrable on $[0,\infty)$. Besides, the other solution may or may not lie in that Lebesque space. These situations are known in the literature as limit-point and limit-circle cases [1] and the results have been introduced with the help of the geometric properties of the combinations of the independent solutions of Equation (1). After this important work, a lot of authors have introduced several results on this type of equation or on the similar equations including difference and dynamic equations [2,3].

In this presentation, we set the Weyl theory for the fractional Sturm–Liouville equation. For this purpose, Caputo and Riemann-Liouville fractional operators were used having the order is between zero and one [4].

Keywords Weyl theory · Fractional differential equation · Spectral analysis

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Approximation by a new modification of Bernstein-Durrmeyer operators

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Abstract

This presentation will be centered about the Durrmeyer-type generalization of the modified Bernstein operators. Then we calculate central moments and mention the approximation properties of the constructed operators. After that, we give the rate of convergence by the help of modulus of continuity, with the help of functions from Lipschitz class and by using Peetre-$\mathcal{K}$ functionals.

Keywords Bernstein-Durrmeyer operators · Rate of convergence · Modulus of continuity

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ON QUATERNIONS WITH GAUSSIAN ORESME NUMBERS

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ABSTRACT

At this paper, we consider Gaussian Oresme numbers. Then we examine some spectacular properties of quaternions with Gaussian Oresme coefficients.

***K*eywords** Binet Formula · Oresme Numbers · Generating Function · Cassini identity · Catalan identity

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EXISTENCE AND MULTIPLICITY OF POSITIVE STEADY STATES FOR CLASSES OF REACTION DIFFUSION EQUATIONS

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ABSTRACT

We study positive solutions to steady state reaction diffusion equations of the form:

$$\begin{cases} -\Delta u = \lambda f(u); \;\; \Omega, \\ \frac{\partial u}{\partial \eta} + \mu(\lambda) u = 0; \;\; \partial\Omega, \end{cases}$$

where $\lambda > 0$, Ω is a bounded domain in $\mathbb{R}^N$; $N \geq 1$ with smooth boundary $\partial\Omega$, $\frac{\partial u}{\partial \eta}$ is the outward normal derivative of u, $\mu \in C([0,\infty))$ is strictly increasing such that $\mu(0) \geq 0$ and $f \in C^2([0, r_0))$ with $0 < r_0 \leq \infty$. If $r_0 < \infty$ we assume $f \in C^2([0, r_0])$ with $f(r_0) = 0$ and $f(s) \leq 0$ for $s \in (r_0, \infty)$, and if $r_0 = \infty$ we assume $\lim_{s\to\infty} f(s) > 0$ and $\lim_{s\to\infty} \frac{f(s)}{s} = 0$ (sublinear at ∞). Note here that the parameter λ influences both the equation and the boundary condition. We discuss existence, nonexistence, multiplicity and uniqueness results for the cases when (A) $f(0) = 0$, (B) $f(0) < 0$, and (C) $f(0) > 0$. We obtain existence and multiplicity results by the method of sub-super solutions and uniqueness results by comparison principles and a priori estimates.

***K*eywords** Boundary Value Problems, Elliptic Partial Differential Equations, and Mathematical Ecology.

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A Class of Multivariate Orthogonal Functions Associated with Fourier Transforms of Orthogonal Polynomials on the Simplex

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Abstract

In this paper, Fourier transform of r-dimensional orthogonal polynomials on the 3-simplex is calculated and by considering the Parseval's identity a new class of multivariate orthogonal functions is derived.

***K*eywords** Fourier transform · multivariate orthogonal polynomials · Parseval identity · recurrence relation

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Rho-Statistical Convergence of Interval Numbers

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Abstract

Statistical convergence is introduced by Fast (1951) and Steinhaus (1951), independently. In 2017, Cakalli defined rho-statistical convergence for real number sequences where rho is a non-decreasing sequence of positive real numbers satisfying certain conditions. Later on, Aral, Kandemir and Et introduced rho-statistical convergence for sequences of sets. With all this information, in this study we investigate rho-statistical convergence of interval numbers.

***K*eywords** Statistical convergence · rho-statistical convergence · interval numbers

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STATISTICAL CONVERGENCE OF MULTISET SEQUENCES FROM A DIFFERENT PERSPECTIVE

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ABSTRACT

The concept of statistical convergence of real numbers has been extended to the statistical convergence of sequences of sets by Nuray and Rhodes in 2012. A multiset is an unordered collection of objects elements are allowed to repeat and a multiset sequence is a sequence whose domain is the set N of natural numbers and range, a set of multisets. In this case, it can be said that multiset sequences are also a set sequence. In this paper, we discuss the statistical convergence of multiset sequences in the Wijsman sense.

***K*eywords** Statistical convergence · Wijsman statistical convergence · multiset sequences

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On k-Oresme Numbers with Negative Indices

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Abstract

Horadam sequence is the most commonly used sequence with recurrence relation. This sequence is a second-order linear sequence and is defined as $W_{n+2} = pW_{n+1} - qW_n$ with initial conditions $a = W_0, b = W_1$. To obtain new sequences from this sequence, the initial conditions a, b, p, q integers are changed. The most common sequences obtained by this method and widely used in the literature are Fibonacci sequences. The new sequence, described by Nicole Oresme and called the Oresme sequence, is actually a special type of the Horadam sequence. The Oresme sequence is a sequence with rational initial conditions and was first studied by Cook. Later, this sequence was re-examined by Horadam, and the numbers the author worked with actually took their place in the literature as a generalization of the Oresme numbers. In this study, we considered and analyzed the k-Oresme numbers with the negative indices. We obtained equalitiesy and identities by using different powers of the matrix defined for k-Oresme numbers. Considering finite sums, we also examined the sums of alternating sum, odd and even terms. We have given some combinatorial equations with the help of the determinant and trace of the matrix defined here.

***K*eywords** Horadam sequence · Oresme numbers · k-Oresme numbers

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k-Oresme Polynomials and their Derivatives

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Abstract

In the Horadam sequence shown as $\{W_n\}_{n\geq 0} = W_n(W_0, W_1; p, q)$, different sequences are obtained by choosing the initial conditions p and q integers differently, and these sequences have an important place in number theory. One of these sequences that has been defined and studied recently is the Oresme number sequence defined by Nicole Oresme. The k-Oresme numbers, which are a generalization for Oresme numbers, were defined and studied by Cook. In this study, we defined and studied k-Oresme polynomials using k-Oresme numbers ann Oresme polynomials. We examined the derivatives of k-Oresme polynomials. By studying these derivatives we have obtained, we have given some important equations and identities involving derivatives.

The formula that gives the general term in the k-Oresme polynomial sequence and known as the Binet formula is given. Using this formula, different and new equations are also given. Then, a square matrix containing the terms of the k-Oresme polynomials is defined and the relationship between the k-Oresme polynomials is given by using the various powers of this matrix. By utilizing the powers of this newly defined matrix, some new identities have been obtained with the components of the matrix. Considering the finite sums of the k-Oresme polynomials, some combinatoric equations are obtained from sums of alternatig sum, odd an deven terms. By considering the derivatives of k-Oresme polynomials combinatorial identity and equations are given. For real x values and positive n values, a new recurrence relation is obtained by taking the derivative of the recurrence relation of k-Oresme polynomials. By using the recursion relation obtained, the identity giving the relationship between the polynomials and their derivations for $n \geq 2$, is given and proved. Finally, some finite sums of k-Oresme polynomials are considered, and equations showing the relationship between derivatives are obtained.

***K*eywords** Horadam sequence · Oresme numbers · k-Oresme numbers · Oresme polynomials

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ON NEW FAMILIES OF BICOMPLEX JACOBSTHAL NUMBERS WITH q-INTEGER COMPONENTS

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ABSTRACT

In this work, quantum calculus plays an important role in physics, number theory and other areas of mathematics. Studies on quantum calculus have increased significantly in physics and mathematics applications. Inspired by these studies, we defined a bicomplex q-sequences with coefficients from Jacobsthal and Jacobsthal Lucas number sequences with components containing q-quantum integers and examined some of its properties. We gave some arithmetic operations of these numbers and the relations between their conjugates. We also examined generating function and Binet formula for newly defined sequences. Using Binet formula, we gave some important identities such as Cassini and Catalan identities. Also, we studied q-sums that make working with special number sequences easier.

Keywords Bicomplex numbers · Quantum integers · Recurrence relations · Jacobsthal numbers

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QUANTUM CALCULUS APPROACH TO THE DUAL BICOMPLEX JACOBSTHAL NUMBERS

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ABSTRACT

A dual number is defined as an ordered pair of real numbers associated with the real unit and the dual unit ϵ. In this study, we benefited from dual bicomplex numbers and quantum analysis studies, which were previously defined with the help of dual numbers. We defined Jacobsthal and Jacobsthal Lucas dual bicomplex number sequences whose coefficients are associated with q-integers. Then, we gave some basic properties of the newly defined number sequences and their relations with each other. We also derived the generating functions and Binet formulas for these sequences. In addition, using these formulas, we gave important identities such as Vajda, Honsberger and d'Ocagne identities which provide the elements of the defined sequences.

Keywords Dual numbers · Bicomplex numbers · Quantum integers · Jacobsthal numbers

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EXTRAPOLATED IMEX RUNGE-KUTTA METHODS TO SOLVE NONLINEAR PARABOLIC PDES

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ABSTRACT

***K*eywords** Implicit–Explicit methods · Nonlinear Parabolic PDEs · Numerical stability

Traditionally classical explicit methods have not been used to solve nonlinear parabolic PDEs due to their stability limitations and the stiffness of the resulting system of ODEs. However, implicit methods require solving very large systems of nonlinear equations at each step.

In this paper extrapolated alternate direction IMplicit-EXplicit (IMEX) are developed through the IMEX$-\theta$ method and their consistency and stability are analyzed. The IMEX$-\theta$ method

$$y^1_{n+1,k} = y_n + kF(t_n, y_n) + (1-\theta)kAy_n + \theta kAy^1_{n+1,k} \tag{20}$$

is a well-known first-order scheme for the semi-discretised system of ODEs:

$$y' = G(t, y) = Ay + F(t, y), \qquad y(t_0) = y_0. \tag{21}$$

IMEX$-\theta$ method is 0-stable whenever $\theta \geq \frac{1}{2}$ and all the eigenvalues of the Jacobian have negative real part ($Re(\lambda_i) \in \mathbb{R}^-$). Additionally the code is very easy to implement if the matrix A is, for example, tridiagonal.

Since extrapolated IMEX depend on a parameter A, we consider the following test problems and according definitions: to study the 0-stability of an IMEX method, the standard Dahlquist's test problem is employed

$$w'(t) = \lambda w(t), \tag{22}$$

where the parameter $\lambda \in \mathbb{C}$ is the parameter chosen in the IMEX method.

Regarding the A-stability of the methods, since our problem is non-linear, it is not easy to choose an optimal λ. Thus, we can consider that $A = \lambda \in \mathbb{C}$, $F(t, w) \sim \mu w \neq 0$. Hence, the test equation for the absolute stability is

$$w'(t) = (\lambda + \mu)w(t), \tag{23}$$

and the parameter in the method is just λ.

Definition: A method is said to be 0(α)-stable if the sector

$$S_\alpha = \{z/\arg(-z) < \alpha, z \neq O\}$$

is contained in the 0-stability region.

Definition: A method is said to be totally 0-stable if the sector $S_{90^\circ} = \mathbb{C}^-$ is contained in the 0-stability region.

In this work it is demonstrated that the second-order extrapolated IMEX methods are totally 0-stable for $\theta \geq \frac{2}{3}$. As for the third-order extrapolated IMEX methods whenever $\theta \geq 0.73$, they are 0(α_θ)-stable, with $\alpha_\theta \geq 89°$. A-stability regions are also studied.

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A Survey On Slant Ruled Surfaces

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Abstract

In this study, we define the slant ruled surface generated by the natural lift curve, which is the special curve obtained by the endpoints of the unit tangent vectors of the main curve, in $\mathbb{R}^3$. Then, we denote that the quaternion product of quaternionic operator whose scalar part is a real parameter and vector part is a curve in $\mathbb{R}^3$ and a spherical striction curve represents a slant ruled surface in $\mathbb{R}^3$ if the vector part of the quaternionic operator is perpendicular to the position vector of the spherical striction curve. Also, we examine the slant ruled surface by using this quaternionic operator. Finally, we give some illustrative examples.

Keywords Natural lift curve · Slant ruled surface · Tangent bundle of unit 2-sphere

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ON ROOTS OF SOME QUATERNIONIC POLYNOMIALS

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ABSTRACT

A monic quaternion polynomial of degree n is defined by $p(x) = x^n + q_{n-1}x^{n-1} + \cdots + q_0$ in the quaternion indeterminate x, where $q_0, q_1, \cdots, q_{n-1}$ are quaternions. In number theory, starting with the Hilbert's 10th problem, finding solutions of equations where the solutions are restricted to the set of integers has received considerable attention. Conics whose equations are satisfied by pairs of successive terms of the Fibonacci and Lucas sequences are also found interesting from many researchers. Inspired by this thought, we consider some quadratic quaternionic polynomials which have generalized Fibonacci and Lucas quaternions pairs of roots. We use two methods to find the roots. One of them is the Niven's algorithm, which is based on the norm and trace of a quaternion and proves the existence of a quaternion root of polynomials over real quaternions. The starting point for this method is the well-known fact that every quaternion q satisfies a second order equation with real coefficients $x^2 - tx + n = 0$, where t is the trace and n is the norm of q. After some computations, Niven has got two real equations for trace and norm of degree $2n-1$. Thus it is proven that the real solutions of these equations give the trace and the norm of the zeros of the unilateral quaternion polynomial $p(x)$, and the inverse is also true. But finding trace and norm using these equations is quite impractical. Thus the information about the trace and the norm of the root will be obtained by the companion matrix, which is represented by a matrix with complex entries, associated to the polynomial. The other method for finding the roots that we use is given in [14], where the real quadratic form matrices are defined, which are used to construct a simple equivalent real quadratic form of monic quaternionic quadratic polynomials.

Keywords Quaternions · Quaternionic polynomials · Real quadratic form · Companion quaternionic matrix

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THE EXTENDED EXPONENTIAL-WEIBULL ACCELERATED FAILURE TIME MODEL WITH APPLICATIONS TO CANCER DATA SET

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ABSTRACT

To model time-to-event data, the Weibull, log-logistic, and log-normal distributions are commonly utilized. Only monotone hazard rates are accommodated by the Weibull family, although log-logistic and log-normal are extensively employed to describe unimodal hazard functions. We need more flexible models that can incorporate both monotone and non-monotone hazard functions since lifespan data with a wide variety of features is becoming more widely available. The extended Exponential-Weibull distribution model, for example, not only supports monotone hazard functions but also allows for bathtub and unimodal shape hazard rates. In univariate study of time-to-event data, this distribution has shown a lot of promise. Many research, on the other hand, are primarily concerned with determining the link between the time it takes for an event to occur and one or more covariates. In time-to-event analysis, this leads to the examination of survival regression models, which may be expressed in a variety of ways. Formulating models for the accelerated failure time family of continuous distributions is one such method. The Weibull, log-logistic, and log-normal distributions are the most widely utilized for this purpose. In this paper, we show that the extended exponential-Weibull distribution is closed under the accelerated failure time framework. Then, using the maximum likelihood approach, we build a survival regression model based on the extended Exponential-Weibull distribution and estimate the model parameters. The performance of the model parameter estimators was demonstrated using a comprehensive Monte Carlo simulation analysis. To demonstrate the applicability of the novel proposed survival regression model, two real-life survival data sets from cancer therapies were used. The simulation and real-world applications show that the proposed model is capable of accurately characterizing various forms of time-to-event data.

***K*eywords** Accelerated failure time model; cancer data; the extended Exponential-Weibull distribution; survival analysis; maximum likelihood estimation; Monte Carlo Simulation; hazard rate.

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Sid Sackson's Mathematical Games

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Abstract

In 2019, Jim Henle published an inspiring paper [1] where he introduced a quadruple of intricate games from Sid Sackson's book 'A Gamut Of Games' [4], as examples of old games without known winning strategies, posing interesting mathematical problems.

In this talk, we will discuss two of these games, namely 'Hold That Line' and 'Cutting Corners'. A winning strategy for 'Hold That Line' was finally found in 2020 and described by the present author in December 2020 of that year in [2]. We will introduce concepts allowing a simple description of the main idea of the strategy. Besides that, the concepts turn out to be useful in the description of strategies for modifications of the original game. 'Hold That Line' is played on a 4×4 field of dots, but we shall provide descriptions of winning strategies for $M \times N$ fields where $M \leq 3$. In case of the field $3 \times N$, we will show how this game corresponds to the game NIM played with two heaps of equal size.

After describing a winning strategy for the neat game 'Hold That Line', we will focus our attention to the perplexing 'Cutting Corners'. While a winning strategy for the former game was found by guessing one 'lucky' move and subsequent case-by-case analysis, 'Cutting Corners' seems too complex to be analyzed without a computer. A winning strategy for this game was obtained using a program written in Wolfram Mathematica in 2022. The program [3] revealed which of the two players has a winning strategy, as well as a few interesting facts as a bonus. We will briefly describe these results, as well as a few observations regarding the game's rules. Besides the original 6-move game by Sid Sackson, we also investigate its variants where the game ends after 4, 8 or 10 moves.

The primary goal of this talk is to summarize the obtained results and to introduce the main ideas of the found winning strategies for both games and their modifications.

***K*eywords** Games · Strategy · Hold That Line · Cutting Corners · Sid Sackson

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ICMASE 2022

A Monge-Kantorovich-type Norm on a Vector Measures Space

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Abstract

In this paper, we use the integral introduced in [4] to equip a vector measure space, namely $cabv(L^q)$, where q is a finite number, greater than 1, with a norm, other than the variational norm. The norm that we introduce here will be called the Monge-Kantorovich type norm. We give an example of computing this norm for a measure from $cabv(L^q)$. Then, we prove a result which compares the topology given by this norm with the topology given by the variational norm.

Keywords variation of a vector measure · Haar functions, Lipschitz functions · the conjugate of a normed vector space · L^p spaces

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ON VECTOR SPACES AND SOME APPLICATIONS

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ABSTRACT

The term "space" is not easy to understand and maybe considered something complicated. Moreover, in science, vectors can be considered as an arrow with a length and a direction. But in mathematics, the combination of these two words has a different meaning, i.e., a vector space is a set V with two operations: addition of vectors and scalar multiplication; these operations satisfy certain properties. At this paper, we present some interesting and spectacular applications and illustrations of vector spaces.

***K*eywords** Vector Space and Matrices and Hamming Distance

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CONVERGENCE AND ERROR ESTIMATION FOR THE INFINITE SYSTEM OF VOLTERRA–FREDHOLM INTEGRAL EQUATIONS INVOLVING ERDÉLYI-KOBER FRACTIONAL OPERATOR

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ABSTRACT

***K*eywords** This talk originates from the investigation of nonlinear fractional integral equations with Erdélyi-Kober fractional operator. In this work the solution of the fractional Volterra–Fredholm integral equations of the second kind is presented. The proposed method is based on the homotopy perturbation method, which consists in constructing the series whose sum is the solution of the problem considered. The problem of the convergence of the series constructed is formulated and a proof of the formulation is given in the work. Additionally, the estimation of the approximate solution errors obtained by taking the partial sums of the series is elaborated on. Moreover, some examples to illustrate the usefulness of our results. Variational inequality theory contains a wealth of new ideas and techniques. Variational inequality theory, which was introduced and considered in early sixties, can be viewed as a natural extension and generalization of the variational principles. It is amazing that a wide class of unrelated problems, which arise in various different branches of pure and applied sciences, can be studied in the general and unified framework of variational inequalities.

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MAPPINGS ON RINGS WITH IDEMPOTENTS

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ABSTRACT

In this talk, we charactrize rings with nontrivial idempotent to show the matrix representation of Lie derivations on this kind of algebrs. At last, we coclude some proved results on B(X) and triangular algebras

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ICMASE 2022

AMOUD-G FAMILY OF LIFETIME DISTRIBUTIONS: PROPERTIES, HAZARD-BASED REGRESSION MODELS AND APPLICATIONS TO SURVIVAL DATA

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ABSTRACT

In this article, we introduce a new family of lifetime distributions with one extra shape parameter, called the Amoud-G family, based on the cumulative hazard rate function, the well-known concept in survival and reliability analysis. Some fundamental theoretical properties of the new family including cumulative hazard function, hazard rate, retro hazard, distribution function, quantile function, residual life function, moments, entropy, among others are derived. A special case of this new family is introduced by considering Weibull distribution as the baseline distribution called Amoud-Weibull distribution. The model parameters of the Amoud-Weibull distribution are estimated using the maximum likelihood estimation technique. The proposed distribution is used to introduce a new accelerated failure time model, which has been used to produce estimations of its model parameters. The performance of the estimation approach based on the Amoud-Weibull distribution and its hazard-based regression modeling has been examined using Monte Carlo simulation analysis. In addition, four examples of real-life data sets with complete and right-censored observations are examined to show the utility and superiority of the newly proposed distribution over other fundamental distributions and their hazard-based regression models in terms of the baseline Weibull distribution and its accelerated failure time model. Finally, the empirical results show that the proposed family of survival distributions provides more realistic fits than other well-known families in terms of complete and right-censored observations..

***K*eywords** Amoud-G family; · cumulative hazard function; · hazard-based regression models · Weibull distribution; · maximum likelihood estimation; · Amoud-Weibull distribution; · censored data; · Monte Carlo Simulation

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ICMASE 2022

QUANTUM GRAPH REALIZATION OF TRANSMISSION PROBLEMS

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ABSTRACT

Differential operators on metric graphs are referred to as quantum graphs in literature since they are intimately related to various problems in quantum mechanics and physics of complex wave systems. They are widely used in modelling of many problems in physics, chemistry, biology, engineering and nanotechnology etc [4]. Boundary value problems on an interval are the simplest examples of quantum graph models and they are intensively studied due to their use in modelling of physical phenomena. On the other hand, some boundary value problems have discontinuities at some interior points (called points of interaction) in which transmission conditions are imposed. These are called boundary value transmission problems. Such problems with transmission conditions arise in several models including heat and mass transfer problems, diffraction problems, vibrating string problems when the string is loaded additionally with point masses etc. These type of problems need to be handled with new techniques due to the singularity at an interior point. Usually these problems are treated as eigenvalue problems in appropriate Hilbert spaces with some special inner products depending on the transmission conditions [1, 2, 3]. It is well-known that these problems are self-adjoint in the new Hilbert spaces.

In this study, we investigate the relations between boundary value transmission problems and quantum graphs. Firstly, we introduce quantum graph realization of boundary value transmission problems. Namely, we consider a boundary value transmission problem with one point of interaction and describe this well-known problem as a boundary value problem on a metric graph (two-star graph). Secondly, we consider a two-star graph with general self-adjoint vertex conditions and treating this as a transmission problem, we define a new special inner product which makes the problem self-adjoint in a general sense. We show that the usual self=adjointness of two-star graph with self-adjoint conditions is obtained as a special case. This brings a new perspective into self-adjointness of a quantum two-star graph.

Keywords Transmission problems · Quantum graphs · Boundary value problems · Self-adjoint operators

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Malmquist-Takenaka System and Equilibrium Condition on the Unit Disc and Upper Half-plane

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Abstract

In our work we start from the complex Malmquist-Takenaka orthonormal system. The Malmquist–Takenaka (M-T) system is a system of rational functions–products of Blaschke factors-in the Hardy space of unit disc, defined by Malmquist and Takenaka in 1925 [1, 2]. As a special case, the M-T system contains the classical trigonometric system. This system is frequently applied in system identification. The M-T system can be transformed to the upper half plane with the Cayley transform. Earlier, in [4, 3, 5, 6] the discretization of the system was investigated both on the unit disc and on the upper half plane. Based on these results, an analogue of discrete Fourier transform was developed and the discrete versions was applied successfully for compression and representation of human ECG signals [7, 8].

The discretization nodes on the unit circle and on the real line have similar properties: they satisfy an equilibrium conditions and are stationary points of a discrete energy function. The question whether they are discrete energy minimizer configurations was raised in the papers of Pap and Schipp [4, 3], and it was answered positively recently by Gaál, Nagy, Nagy-Csiha and Révész [6].

In a recent paper [9], Fridli and Schipp introduced the dual of the Malmquist-Takenaka system on the unit disc, and proved discrete biorthogonal property on a set of points of the unit disc. Starting from the result of Fridli and Schipp, in our presentation we introduce the dual system of the Malmquist-Takenaka system on the upper half plane. We prove similar discrete biorthogonality results, as it was proved on the disc. We also study the properties of discretization points on the disc and the upper half-plane, and we prove that they satisfy an analogue equilibrium conditions like on the unit circle and the real line. This results was recently published in [10]. We introduce projection operators connected to the system, and examine the properties of it.

***K*eywords** Malmquist-Takenaka system · Blaschke product · Equilibrium condition

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NEUTROSOPHIC MULTI-HYPERGROUPS

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ABSTRACT

In this study, we describe neutrosophic multi-hypergroup which is the generalization of the concept of neutrosophic group. Furthermore, we analyze some operations on neutrosophic multi-hypergroups. Moreover, we focus several outcomes and many illustrations regarding these concepts. Beside, we study on homomorphisms of neutrosophic multi-hypergroups. Also, we investigate normal neutrosophic multi-hypergroups. Finally, we examine neutrosophic quotient multi-hypergroups.

***K*eywords** Multisets · Neutrosophic multisets · Multi-Hypergroups

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SIX SIGMA APPLICATION IN LEATHER TEXTITLE COMPANY

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ABSTRACT

This study was carried out in a company operating in the leather textile industry. The company wants to respond to customer orders in a timely manner, to use its production capacity at the highest level and to minimize the waste in production. In order for the company to reach these demands, 6 Sigma method has been applied. With the gradual development of international trade, companies continue to learn and apply new production techniques in order to maintain their existence and continue to work to ensure their continuity. Thanks to the techniques and methods it contains, 6 Sigma is one of the frequently studied topics in the literature. In this study, all five stages of 6 Sigma, namely Identification, Measurement, Analysis, Improvement and Control, were applied. The study started with the definition of the problem. Problems in quality have been identified and a problem determined by pareto analysis has been addressed. In the second stage, data sets related to this problem were created. Necessary measurement and analysis studies were carried out with these data sets. In the next stage, improvements were made for the solution of the problems and their control was ensured. The company, which was at 3.7 Sigma levels before the work was done, increased to 5 Sigma levels after the work was done. With the benefit of this work for the company, wastage has been significantly reduced and productivity has increased. It has been suggested to apply 6 Sigma and its techniques to other processes with the reduction of waste and increase in efficiency.

***K*eywords** 6 sigma · pareto · industry · productivity

ICMASE 2022

SUBMANIFOLDS OF ALMOST COMPLEX METALLIC MANIFOLDS

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ABSTRACT

Let consider the equation $x^2 - px + \frac{3}{2}q = 0$, where p and q are the real numbers satisfying $q \geq 0$ and $-\sqrt{6q} < p < \sqrt{6q}$. In this case the equation has complex roots as $\sigma^c_{p,q} = \frac{p \pm \sqrt{p^2-6q}}{2}$. The complex numbers $\sigma^c_{p,q} = \frac{p+\sqrt{p^2-6q}}{2}$ are named complex metallic means family in [1]. In particular if $p = 1$ and $q = 1$, then the complex metallic means family $\sigma^c_{p,q} = \frac{p+\sqrt{p^2-6q}}{2}$ reduces to the complex golden mean: $\sigma^c_{1,1} = \frac{1+\sqrt{5}i}{2}$, $i^2 = -1$ which is a complex analog of well-known golden mean [5]. By inspiring from the complex metallic means family almost complex metallic structure and almost complex metallic manifold are introduced in [1].

The aim of our paper is to study the geometry of submanifolds of an almost complex metallic manifold. We give fundamental properties of structure induced on submanifolds.

Keywords Complex metallic means · almost complex metallic structure · almost complex metallic manifold · submanifold

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ICMASE 2022

Hirota Bilinear Method and Relativistic Dissipative Soliton Solutions in Nonlinear Spinor Equations

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Abstract

A new relativistic integrable nonlinear model for real, Majorana type spinor fields in 1+1 dimensions is introduced and gauge equivalence of this model with Papanicolau spin model, defined on the one sheet hyperboloid is established [1]. By means of the so called double numbers, the model is represented also as hyperbolic complex valued relativistic model, in the form similar to the massive Thirring model. By Hirota's bilinear method, an exact one and two dissipative soliton solutions of this model are constructed. From infinite hierarchy of integrals of motion, we calculate first three, physically meaningful integrals for one dissipaton solution and show that the last one represents a particle-like nonlinear excitation, with relativistic dispersion and highly nonlinear mass. By analyzing the system of equations for resonant two dissipaton scattering, we find nontrivial solution, showing fusion and fission of relativistic dissipatons. Asymptotic analysis of exact two dissipaton solution and corresponding Mathematica plots confirm resonant character of our dissipaton interactions, depending on parameters and involving creation and annihilation of one and four virtual resonances, similar to non-relativistic Resonant NLS equation [2].
This work was supporting by BAP project 2022IYTE-1-0002.

***K*eywords** Hirota's method · soliton resonances · Majorana spinor · relativistic dissipaton solution

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MAXIMALLY ENTANGLED TWO-QUTRIT QUANTUM INFORMATION STATES AND DE GUA'S THEOREM FOR TETRAHEDRON

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ABSTRACT

Geometric relations between separable and entangled two-qubit and two-qutrit quantum information states are studied. To characterize entanglement of two qubit states, we establish a relation between reduced density matrix ρ_A and the concurrence $\mathcal{C}$. For the rebit states, the geometrical meaning of concurrence as double area of a parallelogram is found and for generic qubit states it is expressed by determinant of the complex Hermitian inner product metric $G_{ij} = \langle c_i|c_j\rangle$ in Hilbert space $C^2, \mathcal{C} = 2\sqrt{\det \rho_A}$, where density matrix ρ_A coincides with the inner product metric, $\rho_A = G^T$ [1]. In the case of generic two-qutrit state

$$|\Psi\rangle = \sum_{i,j=0}^{2} c_{ij}|i\rangle \otimes |j\rangle$$

for reduced density matrix we find relation

$$tr\rho_A^2 + \frac{1}{2}\mathcal{C}^2 = 1,$$

where concurrence $\mathcal{C}$ expressed by sum

$$\mathcal{C} = 2\sqrt{\sum_{i,j=0}^{2} |\det c_{ij}|^2}$$

of all 2×2 minors of 3×3 complex matrix c_{ij}. For maximally entangled two-retrit state, this relation is just De Gua's theorem or a three-dimensional analog of the Pythagorean theorem for orthogonal tetrahedron areas [2],

$$A_{ABC}^2 = A_{ABO}^2 + A_{ACO}^2 + A_{BCO}^2.$$

Generalizations of our results for arbitrary two-qudit states are discussed. This work was supported by BAP project 2022IYTE-1-0002.

Keywords quantum information · qutrit states · entanglement · generalized Pythagoras theorem · De Gua's theorem

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Approximation of Solutions for Nonlinear Functional Integral Equations

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Abstract

Inquiries into nonlinear functional integral equations offer increasing evidence to the extant literature through replication studies. Correspondingly, this paper deals with the solutions to nonlinear functional integral equations using measure of non-compactness. To this end, we first verify the existence of solutions to the considered equation using the generalized Darbo fixed point theorem. Then, we propose an efficient iterative algorithm to find an approximate solution by employing homotopy perturbation method and Adomian decomposition. Further, the efficiency of the algorithm is asserted with two examples. Finally, an error analysis with the upper bound of errors is presented.

Keywords Measure of non-compactness · Nonlinear functional integral equation · Fixed point theorem · Homotopy perturbation

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Application of Double Kashuri Fundo Decomposition Method to Goursat Problem

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Abstract

Differential equations are equations that exist in the background in all areas of our lives, even if we cannot see them, and have an indispensable place especially in mathematics and engineering. Differential equations are in the background of many inventions from the past to the present. Satellites launched into orbit, aircraft, submarine vehicles, pressure and depth studies, experiments with heat, temperature and phases, and many other machines and studies were created as a result of the solutions of these equations. Linear and non-linear partial differential equations take place in the background of many of these machines and studies. These equations help us mathematically model many problems we encounter in many fields, especially in applied mathematics, physics and engineering. Most models that appear in various fields such as fluid dynamics, chemical reaction-diffusion, shallow water wave propagation and Schrödinger wave equation models are expressed with linear and non-linear partial differential equations. The Goursat problem, also known as a quadratic partial differential equation, is a special type of both linear and nonlinear partial differential equation that includes mixed derivatives. It has a very important place especially in the analysis of physical phenomena and applied sciences. There are some methods developed to solve such problems in the literature. In this type of problems, the use of approaches that combine integral transformation and decomposition method can provide an easy solution. In this study, we are looking for a solution to the Goursat problem, which has a very important place in applied sciences, by combining double Kashuri Fundo transform and Adomian decomposition method, namely double Kashuri Fundo decomposition method (DKFDM), which is a new approach (as thoroughly as we could research) in the relevant literature. This approach uses Adomian polynomials to decompose the nonlinear terms. Using this approach, we reach the solution of the Goursat problem without complex calculations and computational difficulties. The applicability and effectiveness of the method used is also demonstrated by various linear and non-linear problems.

***K*eywords** Goursat problem · Kashuri Fundo transform · Double Kashuri Fundo transform · Adomian decomposition method · Double Kashuri Fundo decomposition method

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KASHURI FUNDO DECOMPOSITION METHOD FOR SOLVING MICHAELIS-MENTEN NONLINEAR BIOCHEMICAL REACTION MODEL

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ABSTRACT

Most of the real-life problems are not linear. Therefore, nonlinear ordinary or partial differential equations are used to model them. These equations are very effective in modeling and solving real-life problems. Many problems that seem complex at first glance are modeled with differential equations and become more understandable. One of these models is the Michaelis Menten nonlinear biochemical reaction model. In 1913, the basic enzymatic reaction model was proposed by Michaelis and Menten to describe enzyme processes is an example of nonlinear ordinary differential equations. This model is one of the simplest and best-known approaches of the mechanisms used to model enzyme-catalyzed reactions and is the most studied. In this model, an enzyme and a substrate combine reversibly to form an intermediate complex which subsequently decomposes into a product, regenerating the enzyme. The transient phase of the reaction occurs within a few milliseconds from the start of the reaction. The transient phase of a reaction is essential, from the biochemical point of view, not only for determining various system parameters such as rate constants but also for distinguishing between different mechanisms of enzyme catalysis. Most of the nonlinear ordinary or partial differential equations do not have any analytical solution and some of them have solved by using numerical techniques. In these works, various methods such as homotopy perturbation method, variational iteration method, Adomian decomposition method, differential transformation method, etc. are used. For this reason, semi-analytical and numerical methods are still being developed. On the other hand, new numerical techniques and/or their combinations with the existing ones are attracted the attention of the researchers interested in computational sciences. In this study, a combined form of Kashuri Fundo transform method with Adomian decomposition method, so-called

Kashuri Fundo decomposition method, is used to find an approximate solution to the Michaelis-Menten nonlinear biochemical reaction model.

***K*eywords** Biochemical reaction model · Kashuri Fundo transform · Inverse Kashuri Fundo transform · Adomian decomposition method · Kashuri Fundo decomposition method

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RBF-FD Solution of Natural Convection Flow of a Nanofluid in a Right Isosceles Triangle under the Effect of Inclined Periodic Magnetic Field

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Abstract

In this study, numerical simulation of natural convection flow of Cu-water nanofluid flow in a right triangular cavity under the influence of an inclined periodic magnetic field is conducted. Two-dimensional and steady dimensionless governing equations are considered in stream function-vorticity form. Radial basis function based finite difference (RBF-FD) method augmented with polynomial terms is utilized. The pertinent parameters Hartmann number, inclination angle and period of periodic magnetic field are observed in numerical results. The more Lorentz force means the more slowness in fluid motion and the less convective transfer as well. Horizontal periodic magnetic field coming through the vertical left wall having a heater suppresses the fluid flow and heat transfer significantly comparing to the other inclination angles of magnetic field. In this situation of magnetic field, period one also has more inhibitive effect than the periods less than one.

***K*eywords** RBF-FD · triangular cavity · periodic magnetic field · Cu-water nanofluid

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From Paths to Vector Fields. Application in Optical Flows in Video Frame Sequences

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Abstract

Every path is realizable as a vector field. A **path** in a X is a continuous from the unit interval to a set of points P in a bounded, simply connected surface S. The surface S is **simply connected**, provided every path h has end points $h(0), h(1) \in P$ and h has no self-loops. Paths either lie entirely on a surface in the planar case or lie on a surface and, possibly, puncture a surface in the non-planar case. Paths that puncture a surface are called cross-cuts. A **cross cut path** (also called an ideal arc [1, §3, p.11]) has both ends in P and path interior in the interior of S. The geometric realization of a sequence of paths with no end path (called a path cycle) is a 1-cycle, *i.e.*, a sequence of edges with no end edge. One of the main results in this paper is a consequence of a result in [2], namely, every surface path cycle as well as every collection of intersecting surface path cycles has a free group presentation, which is an extension of the homotopy geometric realization theorem introduced by J.H.C. Whitehead [4]. Betti numbers derived from free group presentation of surface path cycles lead to the discovery of descriptive proximities [3] of nested cycles in vector fields. An application of this work is a surface path approach in the identification of descriptive proximities of path cycles in video frame sequences in the forward propagation of optical flow (recently introduced by J. Montalbo [6]).

***K*eywords** Betti Number · Cycle · Optical Flow · Path · Proximity · Surface · Vector Field

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LEONARDO-MERSENNE SEQUENCE, BINOMIAL TRANSFORM AND SOME PROPERTIES

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ABSTRACT

Because of the important roles and applications of integer sequences like as Fibonacci sequence, Jacobsthal sequence, Leonardo sequence, and some generalizations of these sequences, in matrix theory, numerical algorithms, combinatorial theory, cryptography, and other branches of computer sciences, many authors have studied these sequences. In this paper we introduce the Leonardo-Mersenne sequence. This sequence is a combination of the sequence of Leonardo numbers and Mersenne sequence. The Binet-like-formula, partial sum, generating function and exponential generating function related to this sequence are represented in this paper. Interesting identities and summation formulas for this sequence are represented. Also, we apply the binomial transform to this sequence. Then we investigate some identities of the binomial transform of the Leonardo-Mersenne sequence and give some identities and some summation formulas of it. In order to highlight the results, some numerical examples are given in this paper.

Keywords Leonardo-Mersenne sequence · Binet Formula · Generating Function · Summation formula · Binomial transform.
Mathematics Subject Classification: 11B37, 11B39, 11B83, 05A15

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ON CERTAIN VERTEX OPERATOR ALGEBRAS

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ABSTRACT

In this talk I will present the concept of vertex operator algebras and show some applications to the representation theory of Lie (super)algebras. The emphasis will be on algebras of Virasoro type.

***K*eywords** Vertex algebra · Virasoro algebra · Intertwining operators

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Social Interactions and Mathematical Competencies Development

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Abstract

At technical universities and other tertiary engineering schools, mathematics represents a precursive unavoidable subject of general natural science preparation for futher study in the specific field, and newcomers attend the elementary courses of higher mathematics in the very first semesters. The just finished academic year 2021/22 revealed big differences in skills of new-coming students due to not only different types of graduated secondary schools but reasoning mainly from the way, how schools and predominantly individual students overcome the distance/on line teaching and learning. Except gaps in math knowledge, we observed hard lack of ability to communicate as well as in other social interactions, the state which was only very slowly returning to the expected level.

The paper deals with collaborative learning as the decisive activitazing learning method of many versions, developing math competency consisting of eight overlapping partial competencies introduced in KOM project: C1: Thinking mathematically, C2: Reasoning mathematically, C3: Posing and solving mathematical problems, C4: Modelling mathematically, C5: Representing mathematical entities, C6: Handling mathematical symbols and formalism, C7: Communicating in, with, and about mathematics, C8: Making use of aids and tools; and three dimensions for specifying and measuring progress.

We present the method in three different situations. 1. We bring into consideration the selection of collaborative activities realized during last decade at Faculty of Mechanical Engineering STU in Bratislava reporting good potential to be suitable for competence oriented training and assessment; 2. We introduce three modes of group work mitigating the negative effects of isolation, we carried out within on line teaching; 3. We discuss the situation at the mathematics course in last semester, showing the activities which help students to recapture communication skills, and increase their mathematical competencies, and successfully complete the course. In addition, all presented outcomes are supplemented with opinions, attidutes and self evaluation of involved students.

***K*eywords** Tertiary engineering education · Mathematical competencies · Social interactions · Collaborative learning

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Dynamical Germ-Grain Models with Ellipsoidal Shape of the Grains for some Particular Phase Transformations in Materials Science

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Abstract

Many engineering materials of interest are aggregate of many small crystals, called the grains of the polycrystal, which are often equiaxed. However, because of processing, the grain shape may become anisotropic; for instance, during recrystallization or phase transformations, the new grains may grow in the form of ellipsoids. Heavily anisotropic grains may result from a process, such as rolling, and they may have most of their interfacial area parallel to the rolling plane.

Thus, regarding ellipsoidal grains, one has two related issues. One is the growth of ellipsoidal grains, allowed to overlap each other or not. The other issue is the nucleation on and of ellipsoidal grains in recrystallization processes. This occurs for example after heavy rolling: the grains are aligned ellipsoids and most of the interfacial area of the grains will be parallel to the rolling plane; therefore, to a first approximation, one may consider that these anisotropic grains may be approximated by random parallel planes. Subsequently a new nucleation takes place on such planes. Therefore, modeling the nucleation of ellipsoids on random parallel planes would be of considerable interest in this case.

Moreover, it is reasonable and it has also been found in experimental works, that the probability of a new nucleus forming very close to another one is likely to be low. Therefore, one might suppose that in some cases there is effectively an "exclusion radius "around each nucleus so that within that radius nucleation cannot occur. From a mathematical point of view, such situation may be modelled by assuming hard-core nucleation processes.

In this talk we present a series of germ-grain models, which are actually random closed set eventually dependent on time, as models for the particular phase transformations mentioned above. In particular we provide results on the mean volume and surface density of the associated transformed region. Moreover we also com-

pare two different hard-core point processes (namely Matern and Strauss hard-core point processes) for modelling nuclei with an exclusion zone around them.

Keywords Birth-and-growth process · Random set · Point process · Phase transformation

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INFLUENCE OF THE COLLABORATION AMONG PREDATORS AND THE ALLEE EFFECT ON PREY IN A LESLIE-GOWER-TYPE PREDATION MODEL

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ABSTRACT

Usually, in the real world, the interactions between predators and their prey are influenced by several behaviors of both prey and predators. Collaboration or cooperation between predators is one of these behaviors, which has received less attention than competition between consumers.

These behaviors are important aspects of the dynamics of food chains or trophic webs. In this work, we will deal with the influence of collaboration between predators to consume (or capture) their prey, which is affected by an Allee effect.

***K*eywords** Bifurcation · functional response · limit cycle · predator-prey model · stability.

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A Fast Algorithm for Inversing a Toeplitz Heptadiagonal Matrix Based on the CL Factorization of a Tridiagonal Matrix

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Abstract

Applications in mathematics involve a numerical computations for inverse of a Heptadiagonal matrices; like boundary value problems, the finite element method also the spectral method and many others... The class of Heptadiaonal matrices is a class of special matrices, other common types of special matrices are Jordan, Frobenius, generalized Vandermonde, Hermite, centrosymmetric, and arrowhead matrices.

In this work we purpose a novel fast algorithm to calculate the inverse of a Toeplitz Heptadiagonal matrix H with:

$$\mathbf{H} = \begin{bmatrix} a & b & c & d & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \alpha & a & b & c & d & \ddots & & & & & \vdots \\ \beta & \alpha & a & b & c & \ddots & \ddots & & & & \vdots \\ \gamma & \beta & \alpha & a & b & \ddots & \ddots & \ddots & & & \vdots \\ 0 & \gamma & \beta & \alpha & a & \ddots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & a & b & c & d & 0 \\ \vdots & & & \ddots & \ddots & \ddots & \alpha & a & b & c & d \\ \vdots & & & & \ddots & \ddots & \beta & \alpha & a & b & c \\ \vdots & & & & & \ddots & \gamma & \beta & \alpha & a & b \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \gamma & \beta & \alpha & a \end{bmatrix}$$

We consider the heptadiagonal matrix as sum of tridiagonal matrix T and other diagonal matrix S to apply the Sherman-Morrison-Woodbury to calculate the inverse

of this matrix. So the matrix H can be written as:

$$H = T + S \tag{24}$$

Where:

$$T = \begin{pmatrix} a & b & 0 & \cdots & & & 0 \\ \alpha & \ddots & \ddots & \ddots & & & \vdots \\ 0 & \ddots & \ddots & \ddots & & & \\ \vdots & & \ddots & \ddots & & \ddots & \\ & & \ddots & \ddots & \ddots & \ddots & 0 \\ & & & \ddots & \ddots & \ddots & b \\ 0 & & & \cdots & 0 & \alpha & a \end{pmatrix} \tag{25}$$

And

$$S = \begin{pmatrix} 0 & 0 & c & d & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & & \vdots \\ \beta & 0 & \ddots & \ddots & \ddots & \ddots & d \\ \gamma & \ddots & \ddots & \ddots & \ddots & \ddots & c \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \gamma & \beta & 0 & 0 \end{pmatrix} \tag{26}$$

Secondly we present a new decomposition of the tridiagonal matrices using the CL factorization to calculate the inverse of these matrices.
As a result the inverse of the heptadiagonal matrix have the form:

$$H^{-1} = L^{-1}C^{-1} - \frac{L^{-1}C^{-1}SL^{-1}C^{-1}}{I + L^{-1}C^{-1}S} \tag{27}$$

And we suggest, two algorithms to compare the running time for each one in goal to define the efficiency of our new decomposition.Our algorithm is tested by MATLAB R2014a. As a consequence, we get some interesting results.

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On The Neimark-Sacker Bifurcation of a Certain Second Order Difference Equation

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Abstract

In this study, we handle the Neimark-Sacker bifurcation of unique equilibrium point of following second order difference equation

$$x_{n+1} = px_n + \frac{q}{x_{n-1}^2},$$

where $p, q \in (0, 1)$, and the initial conditions $x_{-1}, x_0 > 0$. We also present some numerical illustrations in order to verify our theoretical results.

***K*eywords** Difference equations · Neimark–Sacker bifurcation · Stability

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PERFORMANCE OF MACHINE LEARNING METHODS USING TWEETS

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ABSTRACT

Public opinions shared in common platforms like Twitter, Facebook, Instagram, etc. act as the sources of information for experts. Transportation and analysis of such data is very important and difficult due to data regulations and its structure. The pre-processing approaches and word-based dictionaries are used to understand the unprocessed data and make possible the opinions/tweets to be analyzed. Data analysis includes master data mining, machine learning, classifying parameters, clustering and prediction. Classification refers to the process of finding and predicting a model that identifies and distinguishes classes of data. Machine learning algorithms learn from past experience and use a variety of statistical, probabilistic and optimization algorithms to detect useful patterns from large, unstructured and complex data sets. Thus, it performs the process of extracting information from the data. In the literature, there are many studies used machine learning algorithms especially related to covid19 virus (Delizo, 2020; Saba et al., 2020; Madani et al., 2021, Chen et al., 2017). The virus was first identified in December 2019 in Wuhan, China and continued to spread rapidly around the World by negatively impacting infected individuals, the health systems, and the global economy. During the COVID19 epidemic, researches on how the disease spreads has been accelerated, and this has paved the way for new research areas (Gill, 2020). There are many methods in the literature to detect various diseases at an early stage (Candra & Bajpai, 2019; Singh & Bajpai 2019). Our study aims to compare the performance of classification algorithms based on different machine learning methods established to predict individuals with COVID(+) or COVID(-) using the emotions among the tweets obtained from Twitter by text mining procedures. Logistic Regression (LR), Support Vector Machine (SVM), Naive Bayes (NB), Decision Trees (DT), Random Forest (RF), Artifical Neural Networks (ANN), Gradient Boost (GBM) and XGradient algorithms were used to extract the accuracy of model performance of each model for

the detection and identification of the disease related to the COVID19 virus, which has been on the agenda recently.

Keywords Classification · Machine Learning · Accuracy · Partition · Sentiment

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TIMELIKE RULED SURFACES IN $3-$DIMENSIONAL A WALKER MANIFOLD

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ABSTRACT

Lorentz geometry is useful in many research fields, especially in generel relativity. Studying submanifolds of a given ambient space plays a very important role in understanding the geometry of this space. Walker studied the pseudo-Riemann manifolds and obtained a canonical form. He called Walker manifold this pseudo-Riemann manifold which is admitted by null distributions. In other words, the Walker manifolds which are created by parallel distributions are also manifolds derived from Lorentz manifolds. More specifically, Walker manifolds that consist of parallel degenerate disrtibutions are known as strict Walker manifolds and the reviews in our work will be examined on strict Walker manifolds. Walker manifolds has the feature of a structure a canonical form that is obtained in the Walker metric, a new metric g_f^ϵ induced in $\mathbb{R}_1^3$ and defined a new inner product. On the other hand a ruled surface is a surface swept out by a straight line moving along a curve. The structural features of ruled surfaces in the Minkowski space have been extensively studied before and the relationship between the characterization of the surface (spacelike, timelike or lightlike) with the characterization of the principal line and base curve on ruled surfaces has been discussed. Fort his study, the ruled surfaces are studied in a $3-$dimensional Walker manifold. Ruled surfaces in Minkowski space by moving to the Walker manifolds and the structural properties of ruled surfaces in the Walker manifolds were investigated. In addition to, the shape operator, Gaussian, mean, Riemann and sectional curvature of timelike ruled surface are calculated and giving examples for timelike ruled surfaces, the calculating of these curvatures is studied. Considering the degenerate and non-degenerate properties of these ruled surfaces, it is desired to investigate how the curvature of the surface is spacelike, timelike or lightlike affects the curvature. Therefore parallel null distributions and parallel null vector fields will be considered. As a result, this work will be completed by calculating the shape operator of the surfaces, the components of the first fundamental form and second fundamental form, sectional curvature, Gaussian curvature, mean curvature and Riemann curvature for non-degenerate ruled surfaces in $3-$dimensional a Walker manifold.

Keywords Walker manifold · ruled surface · timelike

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ON THE SOLUTION OF AN INTEGRAL GEOMETRY PROBLEM OVER SURFACES OF REVOLUTION

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ABSTRACT

The problems of integral geometry are important from both theoretical and practical point of view, especially in imaging. In this study, the problem of reconstructing a function from its integrals over some families of $n-$dimensional surfaces of revolution in $\mathbb{R}^{n+1}$ is investigated. A Fourier slice identity and a back-projection type inversion formula with a method based on the Fourier and Hankel transforms are obtained. Some numerical implementations of the obtained inversion formulas are provided for the cases of $n = 1$ and $n = 2$.

***K*eywords** Generalized Radon transform · Inversion formula · Surface of revolution

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A Discretization Approach and Inversion of Radon Transform via Fuzzy Basic Functions

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Abstract

Inversion of the Radon transform and its generalizations arise in a variety of applications such as medical imaging, geophysics, optics and radar. In these applications, it is essential to develop efficient reconstruction algorithms. In this work, the problem of approximately inverting the Radon transform by some algebraic iterative methods is investigated by proposing a discrete formulation of the reconstruction problem based on a fuzzy partition with Ruspini condition. The feasibility and semi-convergence behavior of the proposed method is demonstrated by analyzing the effect of noisy data and by comparing the method with the pixel-based algebraic iterative methods and the filtered backprojection method.

Keywords Radon transform · Fuzzy partition · Algebraic Reconstruction Techniques · Simultaneous Iterative Reconstruction Techniques

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On Hybrid Numbers with Gaussian-Mersenne Coefficients

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Abstract

At this paper, we consider hybrid numbers with Gaussian Mersenne coefficients and investigate their some interesting properties such as Binet formula, Cassini, Catalan, Vajda, D'Ocagne and Honsberger identities. Moreover, we illustrate the results with some examples.

Keywords Hybrid Gaussian Mersenne · Generating function · Binet formula

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Further Fixed Point Results for Rational Suzuki Contractions in B-Metric-Like Spaces

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Abstract

In 2012 Wardowski established a new contraction, the so called contraction, and obtained a fixed point result as a generalization of the Banach contraction principle. A consecutive work from many researchers has followed this concept with extensions and generalizations provided in metric spaces and generalized metric spaces, by reducing, removing or weakening some of the conditions of definition for this type of contractions. This done work include certain interesting results that describe and cover a large class of contractive functions in the literature of the fixed point theory. Recently many authors have presented new theorems and corresponding clasical results in a broad set such as b-metric and b-metric-like spaces. Huaping et. al. introduce the concept of convex F-contractions and prove some unique fixed point theorems for both continuous and discontinuous mappings. For the fixed point theory in b-metric-like spaces, that is more widely used and associated with applications to show existence and finding solutions for integral equations, impulsive differential equations and fractional differential equations and other engineering problems can see the extended refered literature. In this work we introduce the generalized Suzuki rational (s, q, F)-contractions and investigate some fixed point theorems for these types of contractions in the structure of b-metric-like spaces. By defining the class of F-Suzuki rational contractions, we extend and generalize some previous existing type of contractions such as Jaggi and Gupta contractions. Also we use a reduced number of the axiomatic conditions for contractions, by mean of an important lemma in this setting. The presented theorems extend and generalize the latest fixed point work for contractive mappings in metric and generalized metric spaces. The abstract part is not sufficient enough to describe the background of the proposed work. More background on b-metric-like spaces, F-contraction, and recent work on applicative approach of fixed point theory should be provided by the authors through the presentation.

***K*eywords** F-contraction · Suzuki (s, q, F)-contraction · b-metric-like · fixed point.

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Ayman Shehata, (Egypt), Listener

Ayşe Torun, (Türkiye), Oral Presentation

Bengisen P. Geridonmez, (Türkiye), Oral Presentation

Boutaina Talibi, (Morocco), Oral Presentation

Buse Ingenc, (Türkiye), Listener

Cemil Buyukadalı, (Türkiye), Oral Presentation

Cristina Caridade, (Portugal), Oral Presentation

Daniela Richtarikova, (Slovakia), Oral Presentation

Elena Villa, (Italy), Oral Presentation

Elmira Yu. Kalimulina, (Russia), Oral Presentation

Emel Karaca, (Türkiye), Oral Presentation

Erkan Taşdemir, (Türkiye), Oral Presentation

Esra Güldoğan Lekesiz, (Türkiye), Oral Presentation

Eva Morais, (Portugal), Oral Presentation

Faryal Ch, (Pakistan), Listener

Fatma Aybike Çuha, (Türkiye), Oral Presentation

Fouad Mohammad Salama, (Palestine), Listener

Gokhan Mutlu, (Türkiye), Oral Presentation

Gonca Kızılaslan, (Türkiye), Oral Presentation

Gordan Radobolja, (Crotia), Oral Presentation

Hafize Gumus, (Türkiye), Oral Presentation

Halime Altuntas, (Türkiye), Listener

Ibrahim Erdal, (Türkiye), Oral Presentation

İlkay Tuğ, Eskişehir Technical University, (Türkiye), Oral Presentation

Ion Mierlus-Mazilu, (Romania), Oral Presentation

İlker Akkuş, (Türkiye), Listener

Jesus Martin Vaquero, (Spain), Oral Presentation

Jindřich Michalik, (Czach Republic), Oral Presentation

Kadir Kanat, (Türkiye), Listener

Kastriot Zoto, (Albania), Oral Presentation

Lakshmi Narayan Mishra, (India), Oral Presentation

Maria Emília Bigotte de Almeida, (Portugal), Oral Presentation

Megrous Amar, (Algeria), Oral Presentation

Mehmet Emin Elmas, (Türkiye), Listener

Mehmet Niyazi Çankaya, (Türkiye), Oral Presentation

Mehsin Jabel Atteya, (Iraq), Oral Presentation

Melek Sofyalıoğlu, (Türkiye), Listener

Merve Kısakol, (Türkiye), Listener

Miguel Ángel González León, (Spain), Oral Presentation

Mustafa Özkan, (Türkiye), Oral Presentation

Mücahit Akbıyık, (Türkiye), Oral Presentation

Mükerrem Bahar Baskır, (Türkiye), Oral Presentation

Nalin Fonseka, (USA), Oral Presentation

Nechifor Ana-Gabriela, (Romania), Oral Presentation

Nikita Zena van Gils, (Switzerland), Listener

Oktay Pashaev, (Türkiye), Oral Presentation

Omer Mohamed Egeh, (Somalia), Oral Presentation

Remus Boboescu, (Romania), Oral Presentation

Roudy El Haddad, (France), Oral Presentation

Sachikanta Nanda, (India), Listener

Sandra Ricardo, (Portugal), Oral Presentation

Selçuk Özcan, (Türkiye), Oral Presentation

Selin Erdal, (Türkiye), Oral Presentation

Serhat Yıldırım, (Türkiye), Oral Presentation

Serifenur Cebesoy Erdal, (Türkiye), Oral Presentation

Serkan Onar, (Türkiye), Oral Presentation

Seyyed Hossein J. Petroudi, (Iran), Oral Presentation

Süleyman Aydınyüz, (Türkiye), Oral Presentation

Şule Çürük, (Türkiye), Oral Presentation

Takao Komatsu, (China), Listener

Tane Vergili, (Türkiye), Oral Presentation

Thaer Syam, (USA), Oral Presentation

Tomohiro Sogabe, (Japan), Listener

Tor Kringeland, (Norway), Listener

Vijai Kumar Pathak, (India), Oral Presentation

Vildan Ozturk, (Türkiye), Listener

Yanal Al-Shorman, (Jordan), Oral Presentation

Yıldıray Çelik, (Türkiye), Oral Presentation

Zaki Mahamed Amin, (Egypt), Oral Presentation

Zehra Betül Gür, (Türkiye), Oral Presentation

Zekeriya Ustaoglu, (Türkiye) , Oral Presentationv

Zsuzsanna Nagy-Csiha, (Hungary), Oral Presentation