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# ON THE BI-PERIODIC EDOUARD AND THE BI-PERIODIC EDOUARD–LUCAS NUMBERS

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## ABSTRACT

Recently, several families of sequences of numbers have been studied, such as Fibonacci, Pell, Lucas, Leonardo numbers, and their generalizations. One of these interesting sequences is the Edouard and Edouard–Lucas numbers, $\{E_n\}_{n\geq 0}$ and $\{E_n\}_{n\geq 0}$, defined by the recurrence relation and initial conditions

$$E_n = 7E_{n-1} - 7E_{n-2} + E_{n-3}, E_0 = 0, E_1 = 1, E_2 = 7, \tag{1}$$

$$K_n = 7K_{n-1} - 7K_{n-2} + K_{n-3}, K_0 = 3, K_1 = 7, K_2 = 35. \tag{2}$$

respectively. Relations (1) and (2) can be rewrote as a non-homogeneous relations, given below

$$E_n = 6E_{n-1} - E_{n-2} + 1, E_0 = 0, E_1 = 1,$$

$$K_n = 6K_{n-1} - K_{n-2} - 4, K_0 = 3, K_1 = 7.$$

The Edouard and Edouard-Lucas numbers was explored in the literature (see more in [4] and references there in).

Various generalizations of known number sequences have also been considered. For example, for any real nonzero numbers $a$ and $b$, Edson and Yayenie [2] introduced the generalization of the Fibonacci sequence. The bi-periodic Fibonacci numbers $\{F_n^{(a,b)}\}_{n\geq 0}$ is defined recursively by

$$F_0^{(a,b)} = 1,\ F_1^{(a,b)} = 1,\ F_n^{(a,b)} = \begin{cases} 6F_{n-1}^{(a,b)} + F_{n-2}^{(a,b)}, & \text{if } n \text{ is even} \\ 6F_{n-1}^{(a,b)} + F_{n-2}^{(a,b)}, & \text{if } n \text{ is odd} \end{cases},\ n \geq 2\,. \tag{3}$$

Other works of literature explore this type of generalization, see [1, 3].

When $a = b = 1$, in Equation (3), we have the classical Fibonacci sequence, and for $a = b = 2$, we get the Pell numbers. If we set $a = b = k$ for some positive integer $k$, we come to the $k$-Fibonacci numbers.

In this study, we introduce two new sequences: the bi-periodic Edouard and the bi-periodic Edouard–Lucas numbers.

**Definition 1.** For any two non-zero real numbers $a$ and $b$, the bi-periodic Edouard numbers $\{E_n^{(a,b)}\}_{n\geq 0}$ is defined recursively by

$$E_0^{(a,b)} = 0,\ E_1^{(a,b)} = 1,\ E_n^{(a,b)} = \begin{cases} 6aE_{n-1}^{(a,b)} - E_{n-2}^{(a,b)} + a, & \text{if } n \text{ is even} \\ 6bE_{n-1}^{(a,b)} - E_{n-2}^{(a,b)} + b, & \text{if } n \text{ is odd} \end{cases},\ n \geq 2\,.$$

The first six elements of the bi-periodic Edouard numbers are

| $E_0^{(a,b)}$ | $E_1^{(a,b)}$ | $E_2^{(a,b)}$ | $E_3^{(a,b)}$ | $E_4^{(a,b)}$ | $E_5^{(a,b)}$ |
|---|---|---|---|---|---|
| $0$ | $1$ | $7a$ | $42ab + b - 1$ | $252a^2b + 6ab - 12a$ | $1512a^2b^2 + 36ab^2 - 114ab + 1$ |

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When, $a = b = 1$ we have the classic Edouard numbers.

**Definition 2.** For any two non-zero real numbers $a$ and $b$, the bi-periodic Edouard–Lucas numbers $\{K_n^{(a,b)}\}_{n\geq 0}$ is defined recursively by

$$K_0^{(a,b)} = 3,\ K_1^{(a,b)} = 7,\ K_n^{(a,b)} = \begin{cases} 6aK_{n-1}^{(a,b)} - K_{n-2}^{(a,b)} - 4a, & \text{if } n \text{ is even} \\ 6bK_{n-1}^{(a,b)} - K_{n-2}^{(a,b)} - 4b, & \text{if } n \text{ is odd} \end{cases},\ n \geq 2\ .$$

The first six elements of the bi-periodic Edouard–Lucas numbers are

| $K_0^{(a,b)}$ | $K_1^{(a,b)}$ | $K_2^{(a,b)}$ | $K_3^{(a,b)}$ | $K_4^{(a,b)}$ | $K_5^{(a,b)}$ |
|---|---|---|---|---|---|
| $3$ | $7$ | $38a - 3$ | $228ab - 22b - 7$ | $1368a^2b - 132ab - 84a + 3$ | $8208a^2b^2 - 792ab^2 - 732a + 36b + 7$ |

When, $a = b = 1$ we have the classic Edouard numbers.

In addition, we establish some properties, identities, and recurrence relations of these sequences. The relation with the Balancing and Lucas–Balancing numbers are explored and some identities involving these sequences are provided.

***K*eywords** Balancing sequence · Edouard numbers · Edouard–Lucas numbers

## References

[1] Emre S., and Dursun T., Bi-periodic balancing quaternions. Turkish Journal of Mathematics and Computer Science, 12.2, 68-75, 2020

[2] Marcia E., and Omer Y., A new generalization of Fibonacci sequence and extended Binet's formula. Integers, 9, no. 6, pp 639–654, 2009.

[3] Paula C., and Elen S., A Note on Bi-Periodic Leonardo Sequence. Armenian Journal of Mathematics 16.5, 1-17, 2024.

[4] Yüksel S.; Generalized Edouard numbers. International Journal of Advances in Applied Mathematics and Mechanics, v. 3, n. 9, p. 41-52, 2022.