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NEW DIFFERENCE SETS CONSTRUCTED VIA THE CYCLIC CLASSES METHOD

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ABSTRACT

Difference sets are fundamental objects in combinatorial design theory due to their applications in error-correcting codes, finite geometries, and cryptography. A subset $D \subseteq G$ of a finite group G of order v is called a (v, k, λ)-difference set if every non-identity element of G can be expressed as a difference $d - d'$, with $d, d' \in D$, in exactly λ ways. Constructing such sets remains a central problem in algebraic combinatorics.

In this study, we develop and apply a method called the *Cyclic Classes Method (CCM)* to generate difference sets in additive groups of finite fields. Let q be a prime power and e a positive integer such that $e \mid (q-1)$. The multiplicative group $\mathbb{F}_q^*$ is partitioned into e cyclotomic classes of order e, denoted $C_e^0, C_e^1, \ldots, C_e^{e-1}$. The CCM strategically combines these classes to form candidate subsets in the additive group $(\mathbb{F}_q, +)$, and verifies the difference set condition using group ring identities and character sums.

Using this approach, we construct new families of difference sets with parameters not previously reported in the literature. The method also allows for algorithmic generalization and automated verification using MATLAB, offering a powerful tool for exploring large search spaces in $\mathbb{Z}_q$.

Our results contribute to the existing catalogue of known difference sets and highlight the utility of cyclotomic structures in finite fields. The study includes theoretical proofs supported by computational evidence and offers insights into future directions for algebraic and algorithmic design theory.

Keywords difference sets · cyclotomic classes · finite fields

References

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