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AN EXPLICIT EIGENVALUE DECOMPOSITION OF CIRCULANT TRIDIAGONAL MATRICES

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ABSTRACT

Circulant matrices constitute one of the most extensively studied classes of toeplitz matrices because of their remarkable algebraic properties and their wide range of applications in signal processing, image restoration, coding theory, numerical linear algebra, and the numerical solution of differential and integral equations. A comprehensive treatment of the theory of circulant matrices was given by Davis [1], where their algebraic structure, polynomial representation, relationship with the discrete Fourier transform, and numerous applications were systematically presented. In particular, Davis showed that every circulant matrix can be diagonalized by the discrete Fourier matrix, establishing the foundation for subsequent studies on their spectral properties.

Based on the Fourier representation introduced by Davis, Karner et al. [2] investigated the spectral decomposition of several classes of real circulant matrices, including right circulant, left circulant, skew right circulant, and skew left circulant matrices. They explicitly constructed the corresponding eigendecompositions and singular value decompositions, revealing the relationship between circulant matrices, the discrete Fourier transform, and orthogonal transformations.

Later, Köken and Bozkurt [3] considered a special class of odd-order circulant tridiagonal matrices and obtained explicit formulas for their positive integer powers. Their approach was based on the eigenvalue decomposition of circulant matrices together with trigonometric identities involving Chebyshev polynomials. Subsequently, Öteleş and Akbulak [4] generalized these results by deriving a unified closed-form expression valid for both odd- and even-order circulant tridiagonal matrices. Their formulation considerably simplified the computation of matrix powers and demonstrated the effectiveness of Chebyshev polynomial techniques in the analysis of structured matrices.

More recently, Aliatiningtyas et al. [5] revisited the spectral theory of general circulant matrices and presented explicit formulas for their eigenvalues, determinants, inverses, and invertibility conditions. Their work also emphasized that the eigenvalues of circulant matrices can be efficiently computed through the discrete Fourier transform, providing a computationally attractive framework for spectral analysis.

Motivated by these studies, this paper considers the real circulant tridiagonal matrix

$$A_n = \operatorname{circ}(a_0, a_1, 0, \ldots, 0, a_{-1}).$$

By using the classical Fourier representation, we derive an explicit eigenvalue decomposition of A_n. The resulting eigendecomposition provides an efficient framework for computing integer powers, inverse matrices, matrix functions, and other spectral quantities. Consequently, the obtained results extend several existing studies on circulant tridiagonal matrices and provide a useful tool for both theoretical investigations and computational applications.

***K*eywords** circulant matrix · eigenvalue decomposition · Fourier matrix

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