

2-ABSORBING IDEALS OF COMMUTATIVE SEMIRINGS WITH BIPOLAR FUZZY LOGIC

Serkan Onar^{1,*}, E. Mehmet Ozkan², Ayten Ozkan³

¹Department of Mathematical Engineering, Yildiz Technical University, Istanbul, Turkey

²Department of Mathematics, Yildiz Technical University, Istanbul, Turkey

³Department of Mathematics, Yildiz Technical University, Istanbul, Turkey

ABSTRACT

This study aims to provide a comprehensive understanding of the inherent characteristics, attributes, and operational behaviors of 2-absorbing ideals and 2-absorbing primary ideals of commutative semirings within the framework of bipolar fuzzy logic. We begin by considering the algebraic structure of these ideals and their behavior under the logical conjunction (\wedge) and disjunction (\vee) operations.

In bipolar fuzzy sets, the range of the membership degree is $[-1, 1]$. An element has no bearing on the associated property if its membership degree is 0. If an element's membership degree is within the range of $(0, 1]$, it partially fulfills a hidden counter attribute. Conversely, if an element's membership degree falls between $[-1, 0]$, it partially meets a hidden counter characteristic. These degrees are denoted by $\mu : S \rightarrow [-1, 1]$, reflecting the bipolar nature of the membership function.

Through a detailed exploration of their significance in existing literature, we seek to uncover their distinctive traits and potential applications across various logics. Notably, the bipolar fuzzy logic framework introduces a nuanced perspective on the membership degrees of elements within these ideals, where the functions $\mu^+ : X \rightarrow [0, 1]$ and $\mu^- : X \rightarrow [-1, 0]$ denote the degrees of membership and of non-membership of each element $r \in X$ to set A , respectively, $0 \leq \mu^+(r) - \mu^-(r) \leq 1$ for all $r \in X$.

Additionally, the definition of the Cartesian product for these ideals, $I \times J = \{(i, j) \mid i \in I, j \in J\}$, facilitates a more profound examination of their interrelations. This product allows us to extend properties of single ideals to pairs of ideals, thus enriching our understanding of their interactions and dependencies, thereby paving the way for novel research avenues in this logic. Furthermore, the role of lattice theory in structuring these relationships cannot be overlooked, as the set of 2-absorbing ideals forms a lattice under inclusion.

Lastly, our study encompasses an analysis of these ideals under a semiring homomorphism. Let $\phi : S \rightarrow T$ be a semiring homomorphism. If I is a 2-absorbing ideal of S , we examine the conditions under which $\phi(I)$ remains a 2-absorbing ideal in T within the framework of bipolar fuzzy logic. This investigation reveals deeper insights into the preservation of ideal properties across semiring homomorphisms with bipolar fuzzy logic and underscores the robustness of an application of bipolar fuzzy logic to 2-absorbing ideals within different algebraic frameworks.

Keywords 2-absorbing ideal · 2-absorbing primary ideal · 2-absorbing bipolar fuzzy ideal.

References

- [1] Zadeh, L. A. (1996). Fuzzy sets. In Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh (pp. 394-432).
- [2] Chang, C. L. (1968). Fuzzy topological spaces. Journal of mathematical Analysis and Applications, 24(1), 182-190.

*Corresponding Author's E-mail: serkan10ar@gmail.com

- [3] Rosenfeld, A. (1971). Fuzzy groups. *Journal of mathematical analysis and applications*, 35(3), 512-517.
- [4] Badavi, A., (2007). On 2- Absorbing Ideals of Commutative Rings , *Bull. Austral. Math. Soc.*, 75: 417-429.
- [5] Anderson, D.F. ve Badavi, A., (2011). On n- Absorbing Ideals of Commutative Rings, *Comm. Algebra*, 39: 1446-1672.
- [6] Badavi, A., Tekir, U. ve Yetkin, E., (2014). “On 2- Absorbing Primary Ideals in Commutative Rings”, *Bull. Austral. Math. Soc.* , 51: 1163-1173.
- [7] Darani, A.Y., (2016). “On L- fuzzy 2- Absorbing Ideals”, *Italian J. of Pure and Appl. Math.*, 36: 147-154.
- [8] Darani, A.Y. ve Hashemipoor, A., (2014). L- fuzzy 0- (1- or 2- or 3-) 2-Absorbing Ideals in Semiring, *Annals of Fuzzy Math. And Inform.*, 7: 303-311.
- [9] Lee, J. G., and Hur, K. (2019). Bipolar fuzzy relations. *Mathematics*, 7(11), 1044.
- [10] Massa'deh, M. O. (2017). On Bipolar Fuzzy Cosets, Bipolar Fuzzy Ideals and Isomorphism of Γ - near rings. *Far East J. Math. Sci*, 102(4), 731-747.
- [11] Celik, Y. (2018). On bipolar fuzzy soft graphs. *Creative Mathematics and Informatics*, 27(2), 123-132.