
GENERALIZED ANALYTIC FUNCTIONS IN COMPLEX DOMAINS

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ABSTRACT

Generalized analytic functions constitute an important extension of classical holomorphic functions and provide a powerful framework for the study of elliptic partial differential equations. In this work, boundary value problems for generalized analytic functions are investigated by employing methods from complex analysis and the theory of partial complex differential equations. The study begins with the analysis of inhomogeneous Cauchy–Riemann systems and the derivation of integral representations for their solutions through complex integral operators. These representations are then used to establish connections between holomorphic functions and solutions of more general complex differential equations. Particular attention is devoted to the Dirichlet problem for the first-order equation

$$\frac{\partial w}{\partial \bar{z}} + \lambda w = 0,$$

in bounded simply connected domains with Hölder continuous boundary data. By transforming the problem into an operator equation and applying the contraction mapping principle, existence and uniqueness results are obtained under suitable conditions on the parameter λ . Furthermore, explicit estimates for the corresponding solutions are derived in Hölder spaces. The developed approach is subsequently extended to a second-order partial complex differential equation through its reduction to an associated first-order system. This reduction allows the application of fixed-point techniques and complex integral operator methods to establish solvability results and derive a priori bounds for the solutions. The results demonstrate how classical tools of complex function theory can be effectively combined with functional analytic methods to study boundary value problems for generalized analytic functions and related elliptic equations. The obtained estimates provide a theoretical basis for further investigations of higher-order complex differential equations and their boundary value problems.

Keywords Generalized analytic functions · Dirichlet problem · Cauchy–Riemann equations · complex integral operators

References

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