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TENSOR FORM OF MINARES METHOD FOR GENERALIZED HERMITIAN SYLVESTER QUATERNION TENSOR EQUATION WITH AN APPLICATION

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ABSTRACT

This paper concerns the numerical solution of generalized Hermitian Sylvester quaternion tensor equations and their applications to color video encryption and decryption [1]. A generalized Hermitian Sylvester quaternion tensor equation frequently arises in multidimensional signal and image processing problems. Quaternion tensors effectively represent multidimensional color data [2] by preserving correlations among color channels and structural information in color videos. However, the corresponding quaternion tensor equations are large-scale and computationally expensive to solve, especially when traditional real representation techniques are adopted. Such techniques often destroy the original tensor structure and lead to a significant increase in storage requirements and computational complexity. These issues pose significant challenges for the efficient numerical solution of large-scale quaternion tensor equations.

Motivated by these challenges, we extend the MinAres method [3], which is originally designed for symmetric linear systems, to the generalized Hermitian Sylvester quaternion tensor equation. Using quaternion tensor properties and tensor operations, a tensor form of the MinAres method is developed to solve the considered problems. The proposed method works directly with quaternion tensors, thereby preserving the tensor structure and avoiding dimension expansion. In addition, the proposed method inherits some favorable numerical properties of the classical MinAres method and can be implemented for large-scale problems.

Numerical experiments show that the proposed method achieves comparable residual accuracy with lower computational cost than existing iterative methods. Furthermore, applications to color video encryption and decryption show that the proposed method can recover color video data while preserving the quaternion tensor structure. Comparisons with some existing iterative methods(e.g., the MinRes method [4]) are also provided to further show that the proposed method requires less CPU time and storage while maintaining similar numerical accuracy.

Keywords Quaternion · Hermitain · Sylvester quternion equation · Tensor form · MinAres

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